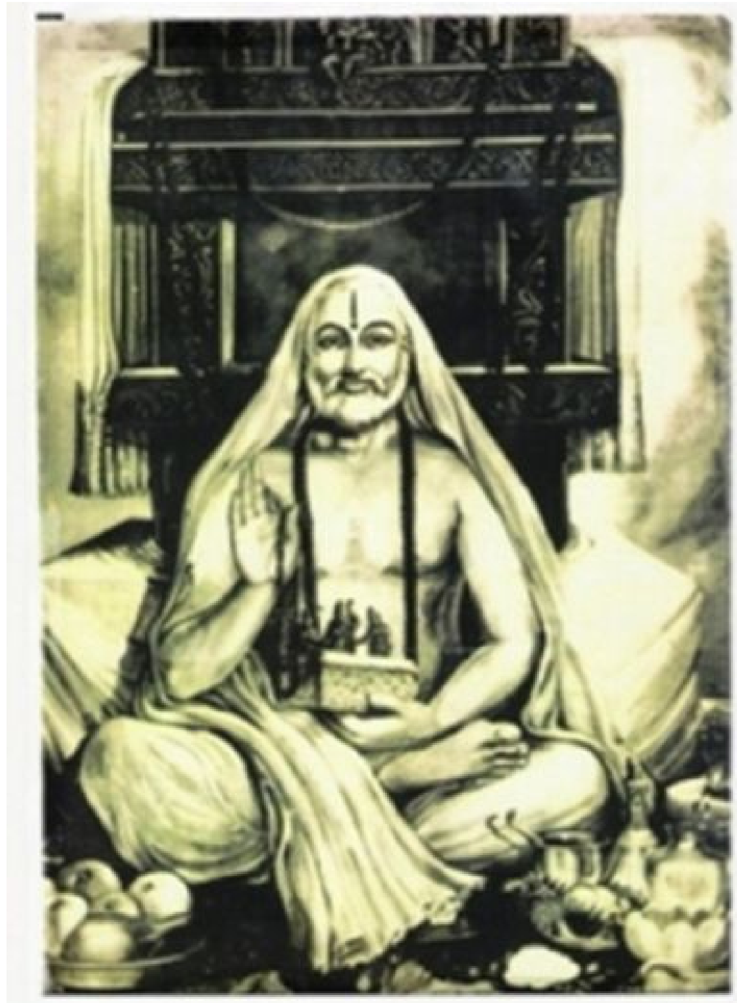

पूज्याय राघवेन्द्राय सत्यधर्मरताय च ।
भजतां कल्पवृक्षाय नमतां कामधेनवे ॥

भारतीयगणितस्य लघुदर्शिनि

श्री वेदव्यासमहर्षिः ॥

National Institute Vedic Sciences

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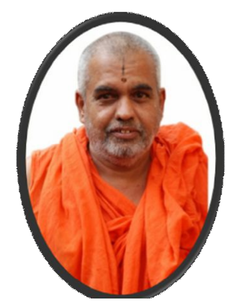
श्री श्री विज्ञाननिधितीर्थ श्रीपादङ्गळवरु ॥



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Relevance of Ancient Knowledge to the present century:

Verification of Aryabhatiya's (AD 499) *Algorithm for finding* 24-*rsines*

Venkatesha Murthy,
Adjunct Faculty,
Centre for Research in Social Sciences and Education.
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\# 52 *Bellary Road, Hebbal, Bangalore* – 560 024.

Introduction: -

Indian Place-value System of base ten with the invention of Zero revolutionized the sciences of the world by helping systematic quantifying of the scientific and Technological concepts.

Aryabhatiya-vyakhya of Bhaskara-I (629 A.D.), *a commentary on Aryabhatiya* (499 A.D.) *of Aryabhata-I explains methods to arrive at the values of* 6-*rsines,* 12-*rsines and* 24-*rsines.*

In general N-*rsines is* '*N values' of r sine* θ_n *which are the products of radius r and 'n number of Sine* θ*' where* θ *is the central angle made by n equal arcs.*

The radius r = 3438 *units considered by Aryabhatiya I is an approximate value of one radian in minutes. The present-day values are also expressed in radian measure. Therefore Aryabhatiya values of rsines almost agree with their present-day values.*

Perimeter of a Circle of diameter 20,000 *units in Aryabhatiya*

Aryabhatiya **of Aryabhata-I has given**

the circumference of a circle of diameter 20,000 units in a s*loka*;

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।

अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥१०॥ **[Refer; 1. (p.71)]**

'Four more than hundred multiplied by eight and increased by sixty-two thousand is a nearer value* (आसन्न) *of the perimeter of a circle of diameter twenty-thousand units'.

[(100 + 4) 8 + 62,000] = 62,832

is a nearer value of the Perimeter of a circle of diameter* 20,000 *units.

***The ratio of perimeter* 62,832 *units to its diameter* 20,000 *units is the Aryabhatiya value of* π;**

$$\pi = \frac{62,832}{20,000} = 3.1416$$

Present-day Radian measure (in minutes) and from Aryabhatiya

***The radian** is the central angle made by an arc of length equal to the radius of the circle.*

***It requires** the ratio of perimeter of the circle to its diameter.*

The ratio of perimeter of a circle to its diameter is named π, because in Greek language 'perimeter' is 'περιμετερ' and π is its first alphabet.

$$\textit{Present-day one Radian} = \frac{180 \times 60}{\pi} = 3437.746771 \ \textit{minutes (app.)}$$

$$\textit{Aryabhatiya one radian} = \frac{180 \times 60}{3.1416} = 3437.738732 \ \textit{minutes (app.)}$$

Aryabhata-I used a circle of radius 3438 units to find N-rsines.

3438 is the measure of 'one radian' in minutes, approximately.

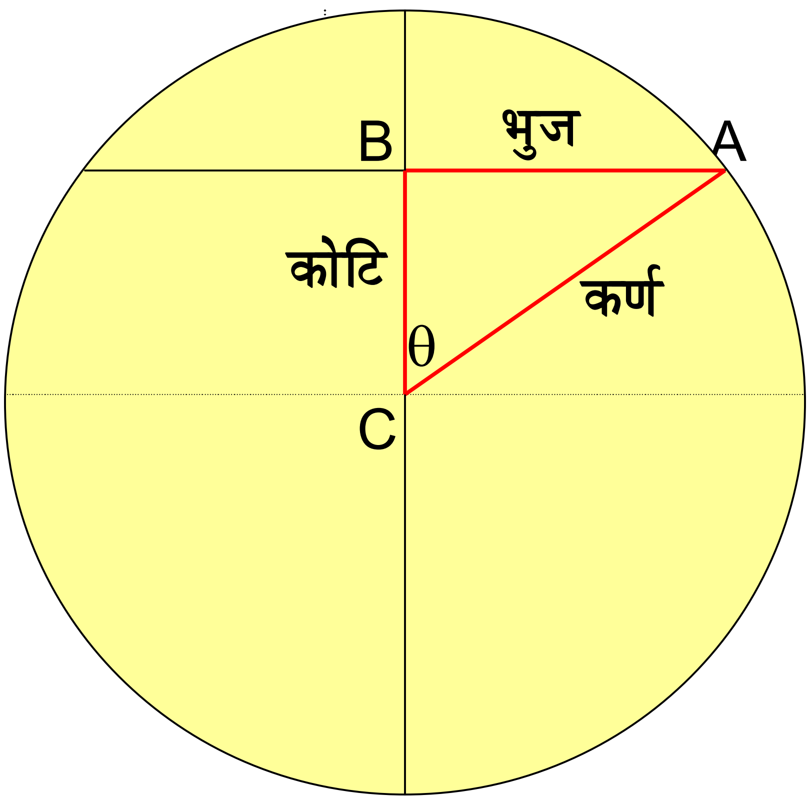
RSINE
in
ARYABHATIYA
(500 AD)

Aryabhata -I

Names of Trigonometrical functions in *Aryabhatiya*

The names of Trigonometrical functions given to the parts of the circle are based on the verses in *Aryabhatiya*.

यश्चैव भुजावर्गः कोटीवर्गश्च कर्णवर्गः सः।



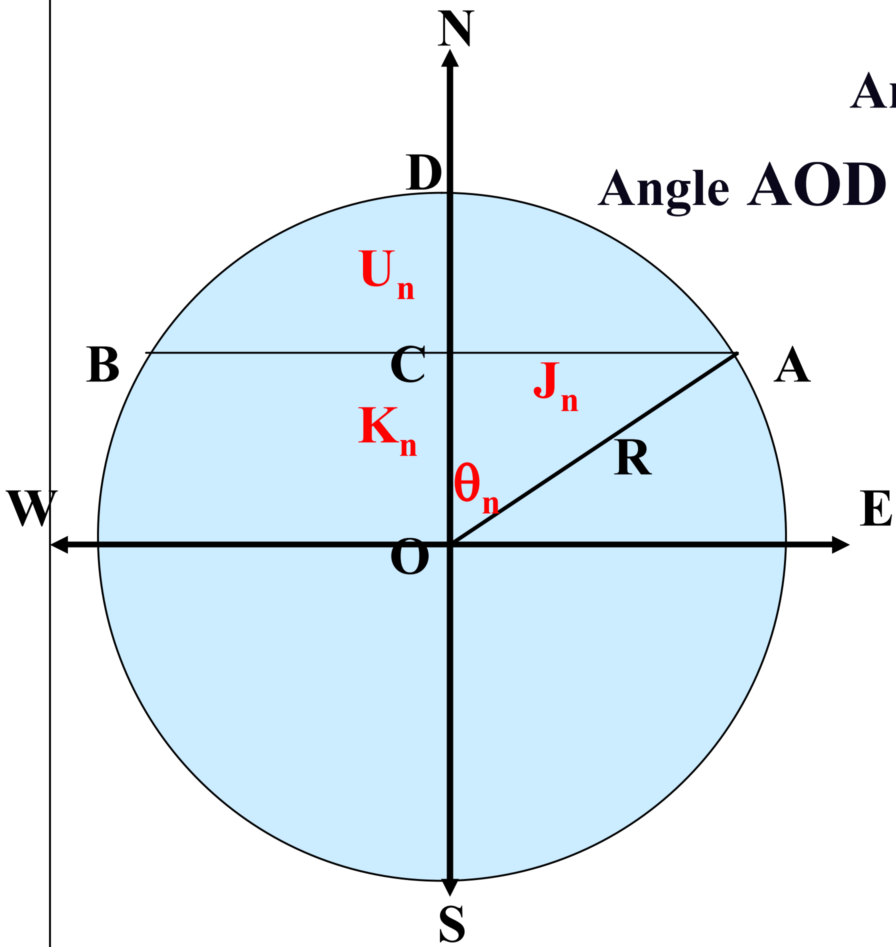
(*In a right-angled triangle*) *the square of the bhuja* (*base of horizontal*) *together with the square of the koti* (*upright or vertical*) *is the square of the karna* (*hypotenuse*)

(भुज)^2 + (कोटि)^2 = (कर्ण)^2

$(AB)^2 + (BC)^2 = (AC)^2$

Trigonometric Functions in Indian Mathematics

[Ref: 'Indian Astronomy; A Source –Book', B V Subbarayappa, K V Sharma, NEHRU CENTRE, BOMBAY (1985) p. 300]



Arc AD is divided into n equal arcs

Angle AOD = θ_n = angle made by n equal arcs

AC = R sin θ_n = J_n
= ज्या of n arcs

OC = R cos θ_n = K_n
= कोटिज्या of n arcs

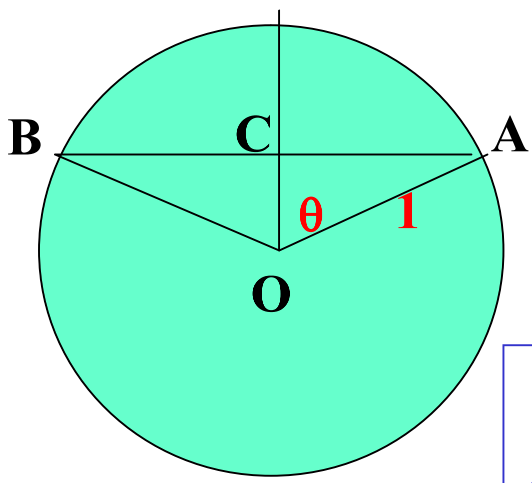
CD = (R - R cos θ_n) = U_n
= उत्क्रमज्या of n arcs

Angles are measured in the clockwise direction from N

Etymology of the term *'Sine'*

[Ref: ***'The Mainstream of Mathematics"* by Edna E Kramer,**
***Oxford University Press, Inc*.**(1951) P.140]

The derivation of our term ***'sine'*** from the ***original Hindu*** is of ***philological interest***.



Aryabhata (500AD) used the word **ज्या** or **जीवा** for **AC**.
[AC = OB sine θ = sine θ]

Arabs translated **जीवा** as ***'gib'*** in ***Arabic.***
Arabic word 'gib' is similar to the word ***'fold'***

Arabic word 'gib' was translated into ***Latin*** as ***'sinus'*** **(fold).**

Later ***'sinus'*** became ***'sine'***.

Geometrical Basis for finding *the desired number of Rsine Values*

- *Aryabhatiya* (AD 499)

परिधे षड्भागज्या विष्कम्भार्धेन स तुल्या ॥९॥

(Sloka 9 in Ganita pada of Aryabhata - I)

The chord of one-sixth of the circumference (of a circle) is equal to its radius.

Bhaskara-I started with
a regular hexagon* inscribed *in a circle of radius* 3438 *units
to find *the desired number of Rsine* values
In his *Aryabhatiya Bhasya* (AD 629).

Radian Measure in *Aryabhatiya*

यदिरडवसुयमलरस (((62,832) मितपरिधेरयुतद्वय (((20,000) व्यासः
खखषड्घन ((21,600) लिप्तातमकप्रिधेष्चक्रस्य को व्यास इति ।
***अयुतद्वयं* (20,000) चक्रकलाभिर्हत्वा रडवसुयमलरस ((62,832) स्य विभज्य**
लब्धं चक्रव्यासः । तदर्धं चक्रव्यासार्हं वस्वाग्निवेदराम (3438) सङ्ख्यम् ।
अनेन व्यासार्धेन शास्त्रीयस्सकलोव्यवहारः ॥

62,832 is the *circumference of a circle* - of *diameter* 20,000 units

21,600 is the *circumference of a circle* - of diameter $\dfrac{20,000 \times 21,600}{62832}$

= 6875.477464

Half of 6875.477464 = 3437.738732 = 3438 (app.)

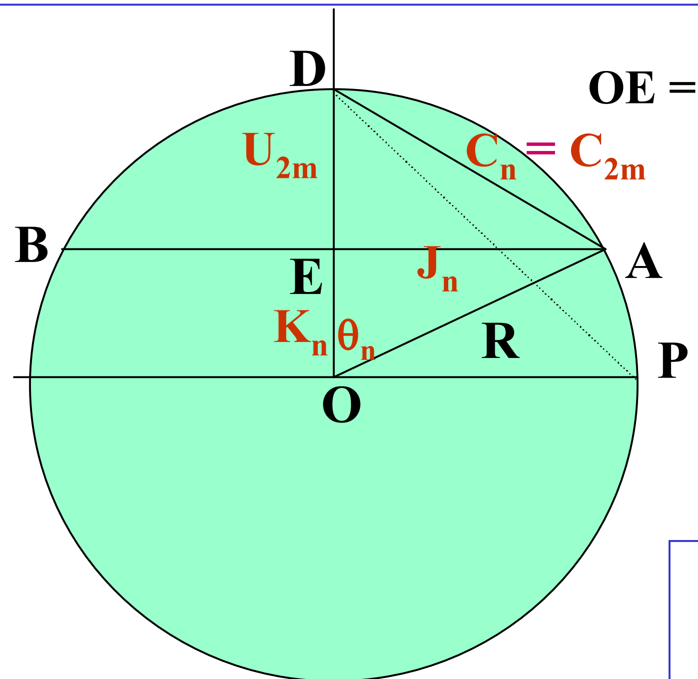
One radian = [180 / 3.1416] = 57. 29577851^{0} (app.)
= 3437.746771 minutes (app.)
= 3438 minutes

R Sin θ from Aryabhatiya (AD 499)

Divide arc PAD into S equal arcs, and arc ADB into 2n equal arcs.

1) AB = Chord of 2n arcs = C_{2n}.
AD = Chord of n arcs = $C_n = C_{2m}$

Arc AD = arc of n arcs. $\angle AOD = \theta_n$
AE = (1/ 2) AB = R sin θ_n = Jya of n arcs = J_n
2) $J_n = (1/2)\, C_{2n}$



OE = $\sqrt{(R^2 - J_n^2)}$ = kotijya of n arcs = K_n = R cos θ_n
[Arc PA = arc of (S - n) equal arcs]
OE =R sin (∠ OAE) = R sin (90 - θ_n) = J_{S-n}
3) $K_n = J_{S-n}$ ∠EAO = ∠AOE = (90 - θ_n)

When n or (S – n) = 2m, an even number
ED = R – K_{2m} = U_{2m} = Arrow of 2m arcs
4) $U_{2m} = R - K_{2m}$

AD = $\sqrt{(J_{2m}^2 + U_{2m}^2)}$ = chord of 2m arcs = C_{2m}
5) $C_{2m} = \sqrt{(J_{2m}^2 + U_{2m}^2)}$ = chord of 2m arcs

C_{2m} is similar to 1. Repeat steps from 2 to 5

I Find [2S/ 3] Rsines, taking $C_{2n} = R = 3438$ (= AB), where 2n = 2S/ 3
II Find [S/ 3] Rsines, taking $C_{2n} = R\sqrt{2} = 3438\sqrt{2}$ (= PD), where 2n = S

Table for six *Rsines* [S = 6]

I Find [2S/ 3] (= 4) *Rsines*, assuming C_{2n} = R = 3438, where 2n = [2S/ 3] = 4 in (1) starting with the chord AB (*a side of the inscribed hexagon*) as C_4

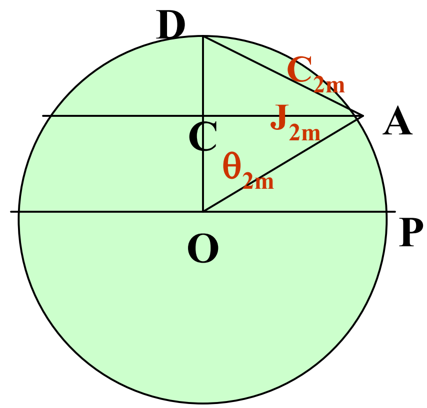
II Find [S/ 3] (= 2) *Rsines*, assuming C_{2n} = R√2 = 3438√2, where 2n =S = 6 in (1I) starting with the chord PD (*hypotenuse of the right angled triangle POD*) as C_6.

1	2	3	4	5	6	7	8
C_{2m}	$J_n = \frac{C_{2n}}{2}$	$K_n = \sqrt{(R^2 - J_n^2)}$	n = (S-2m) $A_{2m} = R - K_{2m}$	$C_{2m} = \sqrt{(A_{2m}^2 + J_{2m}^2)}$	J_{2m}	θ_{2m}	$R\sin\theta_{2m}$
I. C_4= 3438							
C_4= 3438.00	J_2= 1719.00	K_4= 2799.40	A_2= 460.60	C_2=1779.64	J_2= 1719	30°	1719
		J_4= 2977,40			J_4= 2978	60°	2978.40
C_4= 1779.64	J_2= 889.82	K_1= 3320.85			J_1= 890	15°	889.82
		J_5= 3320.85			J_4= 3321	75°	3320.85
II. C_4 = 3438√2 = 4862.07							
C_6= 4862.07	J_3= 2431.03	K_2= 2431.03			J_3= 2431	45°	2431.03
	J_6= 3438				J_4= 3438	90°	3438

$R \sin \theta_{2m}$ from *Aryabhatiya*

To get n or θ_{2m} when S and, one of n and θ_{2m} are known

Arc PD (of quadrant OPAD) is divided into S equal arcs.



Arc AD is divided into 2m equal arcs.

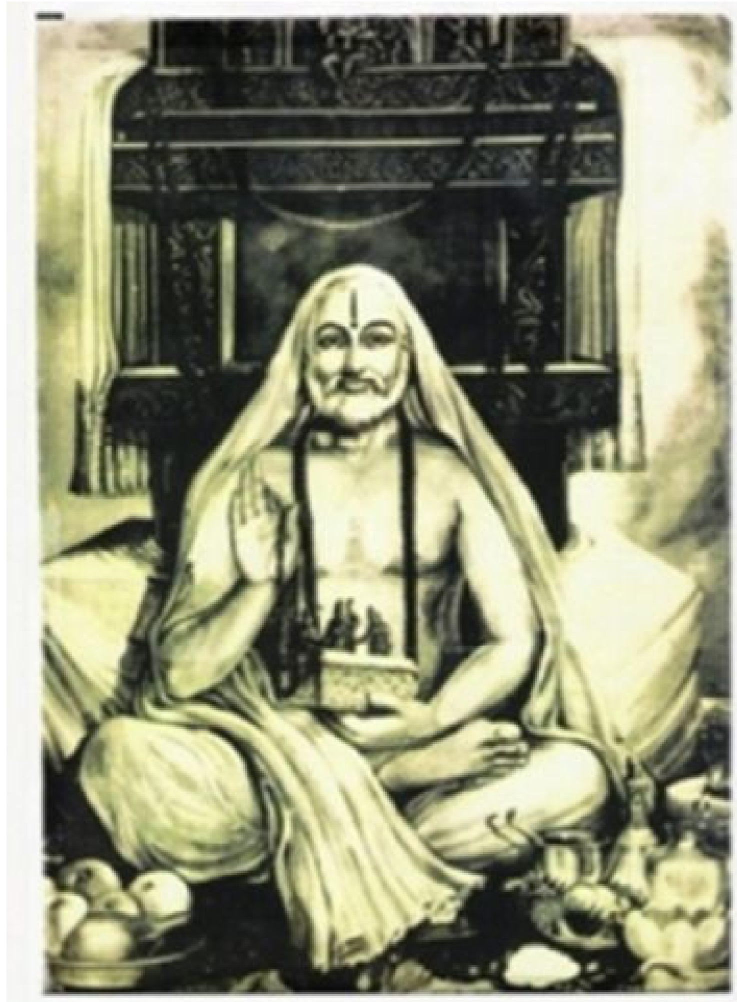
Angle AOD = θ_{2m}

$$\frac{\text{2m, number of equal arcs in Arc AD}}{\text{S, number of equal arcs made in the Arc PD}}$$

$$= \frac{\theta_{2m}\text{, central angle made by 2m equal arcs of Arc AD}}{90°\text{, central angle made by S equal arcs of the Arc PD}}$$

$$\frac{2m}{1} = \frac{\theta_{2m}\, S}{90°} \qquad \frac{\theta_{2m}}{1} = \frac{90° x\ 2m}{S}$$

THANK YOU



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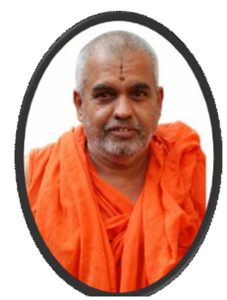

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Relevance of Ancient Knowledge to the present century:

Verification of Aryabhatiya's (AD 499) *Algorithm for finding* 24-*rsines*

***Feb.* 26 - 2012**

Venkatesha Murthy,
Honorary Head, Vedic Math Branch
National Institute of Vedic Sciences,
 # 58, Raghavendra Colony,
Chamarajapet, *Bangalore* – 560 018.

Introduction: -

यथा शिखा मयूराणां नागानां मणयोयथा ।
तद्वद् वेदाङ्ग शास्त्राणां गणितं मूर्धनि स्थितम् ॥४॥

Like the combs of the peacocks and the crest jewels of the serpents, so does the tradition of mathematics knowledge stand at the head of all the tradition of Sciences knowledge forming the auxiliaries of the Vedas. [Refer; 4. (p.27 & 36)]

Indian Place-value System of base ten with the invention of Zero revolutionized the sciences of the world by helping systematic quantifying of the scientific and technological concepts.

Aryabhatiya-Vyakhya of Bhaskara-I (629 A.D.), a commentary on *Aryabhatiya* of Aryabhata-I explains methods to get the values of 6-r*sines*, 12-r*sines* and 24-r*sines*.

In general D-r*sines* is 'n values' of r sine θ_n which are the products of radius r and 'n number of *Sine* θ_n' where θ_n is the central angle made by n equal arcs, n ranging from 1 to D when the arc of the first quadrant is divided into D equal arcs in the circle of radius r = 3438 units. These values exactly agree with their present-day values because radius r = 3438 *minutes is* an approximate value of *a radian.*

The radian is the central angle made by an arc of a circle of radius r whose length is equal to the radius r and it requires the value of the ratio of circumference of the circle of radius r to its diameter.

The ratio of the circumference of a circle to its diameter , which is now known as π, is stated in *Aryabhatiya* of Aryabhata-I thus;

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥१०॥

[Refer; 1. (p.71)]

'Four more than hundred multiplied by eight and increased by sixty-two thousand is a near value (आसन्न) *of the perimeter of a circle of diameter twenty-thousand units'.*

[(100 + 4) 8 + 62,000] = 62,832 is a *nearer* (approximate) *value* of *the perimeter of a circle of diameter* 20,000 *units*.

Perimeter in Greek is περιμετερ and its first letter 'π' is used to denote 'the ratio of the circumference of a circle to its diameter'

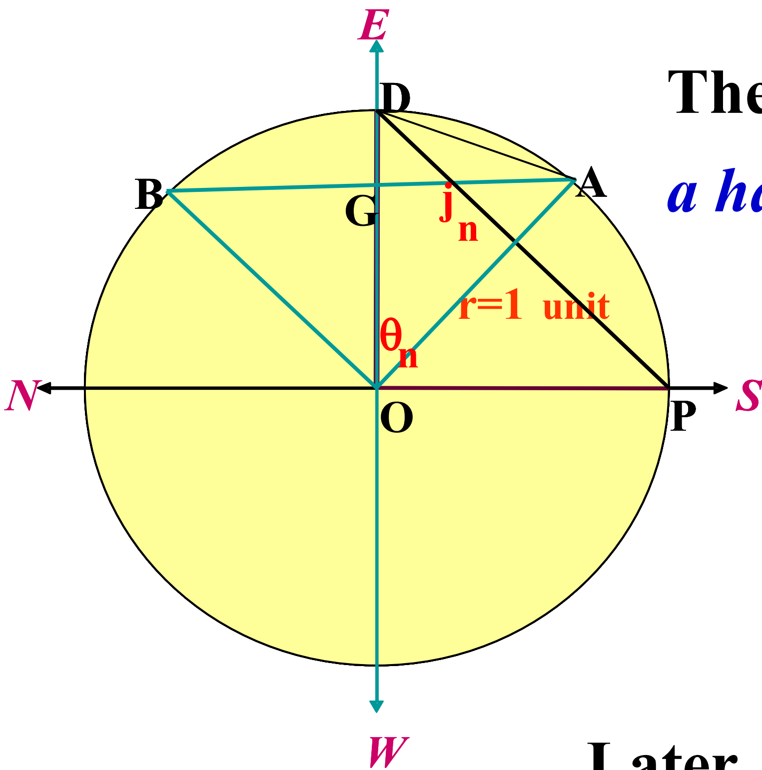
$$\pi = \frac{\textit{circumference of a circle}}{\textit{diameter}} = \frac{62832}{20000} = 3.1416\left(\textit{app.}\right)$$

TRIGONOMETRIC TERMS

in *Indian Mathematics*

Etymology of the term *'sine'*

The term *'sine'* is from the *original Hindu* word ज्या or जीवा



The word ज्या or जीवा is used for GA ;

a half chord in a circle of* radius r =1 *unit.

Arabs* translated ज्या or जीवा as *'gib.

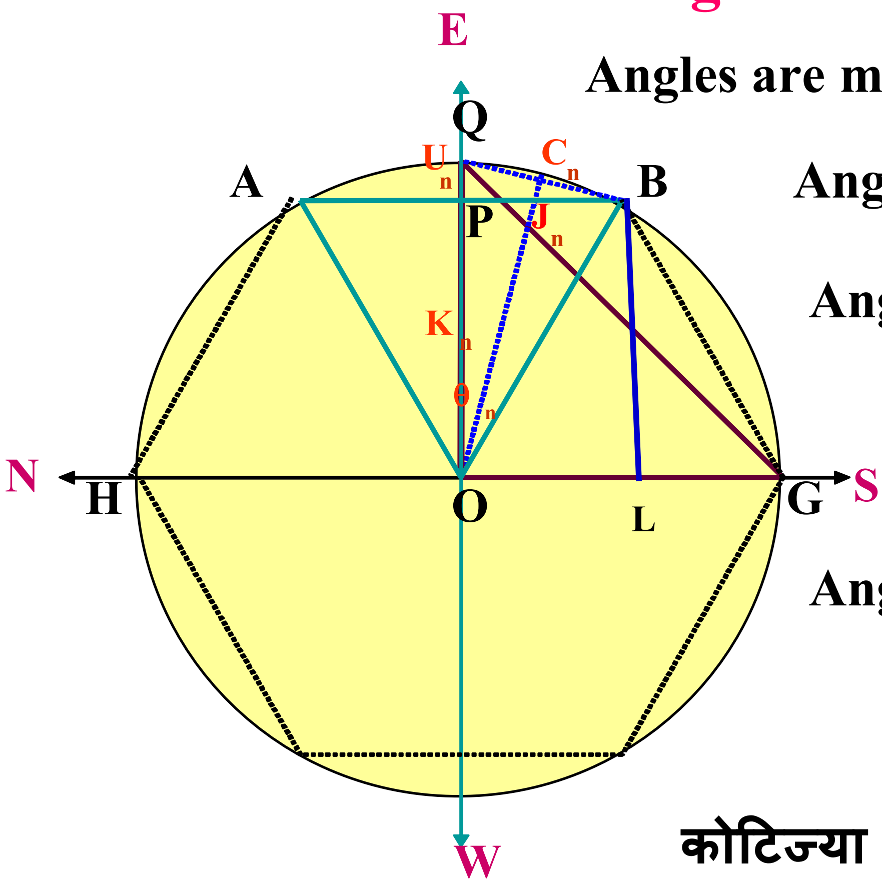
Arabic word* *'gib'* is similar to the word *'fold'

'gib'* was translated into *Latin* as *'sinus'* *(fold).

Later *'sinus'* (ज्या or जीवा) became *'sine'*.

[Ref: *"The Mainstream of Mathematics"* by Edna E Kramer, *Oxford Univ. Press, Inc*.(1951) P.140]

Trigonometric terms in Indian Mathematics



Angles are measured **in the clockwise direction** from **E.**

Angle QOG = 90^o = angle made by **D** equal arcs

Angle QOB = θ_n = angle made by **n** equal arcs

OP = r *cos* θ_n = कोटिज्या of **n** arcs = k_n

Angle BOL = Angle (QOG – QOB) = $(90^0 - \theta_n)$
= Angle made by **(D – n)** equal arcs

OP = LB

कोटिज्या of **n** arcs = k_n = ज्या of (D – n) arcs = $j_{(D-n)}$

$$k_n = j_{(D-n)}$$

PQ = (OQ – OP) = (r - कोटिज्या of **n** arcs) = $(r - k_n)$
= उत्क्रमज्या of **n** arcs = u_n

$$u_n = (r - k_n)$$

Value of (Desired number of) D-*rsines*

In *Aryabhatiya* (500 AD)

Aryabhatiya's (AD 499) Cryptic Algorithm for finding D-r*sines*

Aryabhatiya **of Aryabhata-I (b. 476 AD) gives a method cryptically for finding the values of r*sines* of n equal arcs (r Sin θ_n) = Jya of n equal arcs (J_n).**

To find D-r*sines*

छेद्यकविधिना ज्याऽऽनयनम्

समवृत्तपरिधिपादं छिंद्यात् त्रिभुजाच्चतुर्भुजाच्चैव ।
समचापज्यार्धानि तु विष्कम्भार्धे यथेष्टानि ॥११॥

Divide a quadrant of the circumference of a circle (into as many parts as desired; D).
Then, from (right) triangle and quadrilaterals, one can find as many r*sines* of equal arcs as one likes, for a given radius (3438 *units*).

"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 77]

Aryabhatiya's (AD 499) Cryptic Algorithm for finding D-r*sines*

Bhaskaracarya-I (AD 629) **in his *Aryabhatiya bhasya* elaborately illustrated methods to find 6-*rsines*, 12-*rsines* and 24-*rsines*,**

[6 x $2^{(t-1)}$ where t = 1, 2 or 3].

वसुदहनकृतहुताशनसङ्ख्ये विष्कम्बार्धे कियत्प्तमाणनि ज्यार्धानि ।

r *sines are to be calculated by considering*

***half the diameter of a circle as* 3438.**

As per the rule* अङ्कानाम् वामतोगतिः *'(Place-values of) digits (in a numeral of a number) move (increases) towards left'.

Combination of words* वसुदहनकृतहुताशन *denotes* 3438 *based on words as numerals;

हुताशन	कृत	दहन	वसु
3	4	3	8

"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 78]

Geometrical Basis
for finding the desired number of **rsines**

परिधे षड्भागज्या विष्कम्भार्धेन सा तुल्या ॥९॥

[Refer : *Aryabhatiya, with the commentary of Bhaskara-I and Someswara* :
Edited by K S Shukla, INSA, New Delhi, (1976), (based on p.79 for finding Rsine)]

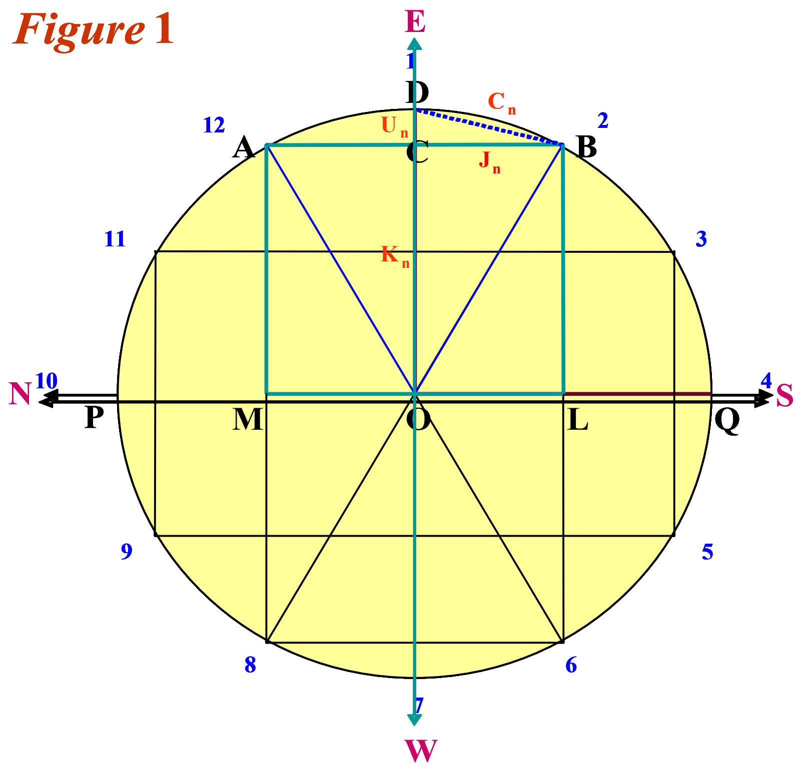
***The chord of one-sixth of the perimeter* (of a circle)**
is equal to its radius.

Therefore, Aryabhatiya Bhasya (AD 629) ***of Bhaskara-I***
has considered a regular hexagon
inscribed in a circle of radius* 3438 *units
***to find the desired number of* (D), rsines.**

Figure to find D-rsines: -

यावत्तावत्प्रमाणपरिच्छिन्नविष्कम्भार्धतुल्येन कर्कटकेन मण्डलमालिख्य तद् द्वादशधा विभजेत् । ते च द्वादशभागा राशय इति परिकल्पाः। अथ द्वादशधा विभक्ते मण्डले पूर्वेण राशिद्वयाग्रावगाहिनीं दक्षिणोत्तरां ज्याकारां लेखां कुर्यात्। एवं पश्चिमभागेऽपि। एवमेव दक्षिणोत्तरभागयोरपि च पूर्वापरयतां ज्यां कुर्यात्। पुनरपि च पूर्वापर दक्षिणोत्तरदिक्षु तथैव च राशिचतुष्टयाग्रावगाहिन्यो लेखाः कुर्यात्। तथा त्र्यश्री (णि) कर्तव्यानि। [Ref. (1). P.78-79]

Figure 1



Meaning: -

***Draw a circle of radius* r (= 3438 *minutes*)**
and divide it into twelve equal parts.
***Consider them as* द्वादशभागा राशय.**

Draw N–S line passing through the centre of the circle. Likewise draw E–W line also.

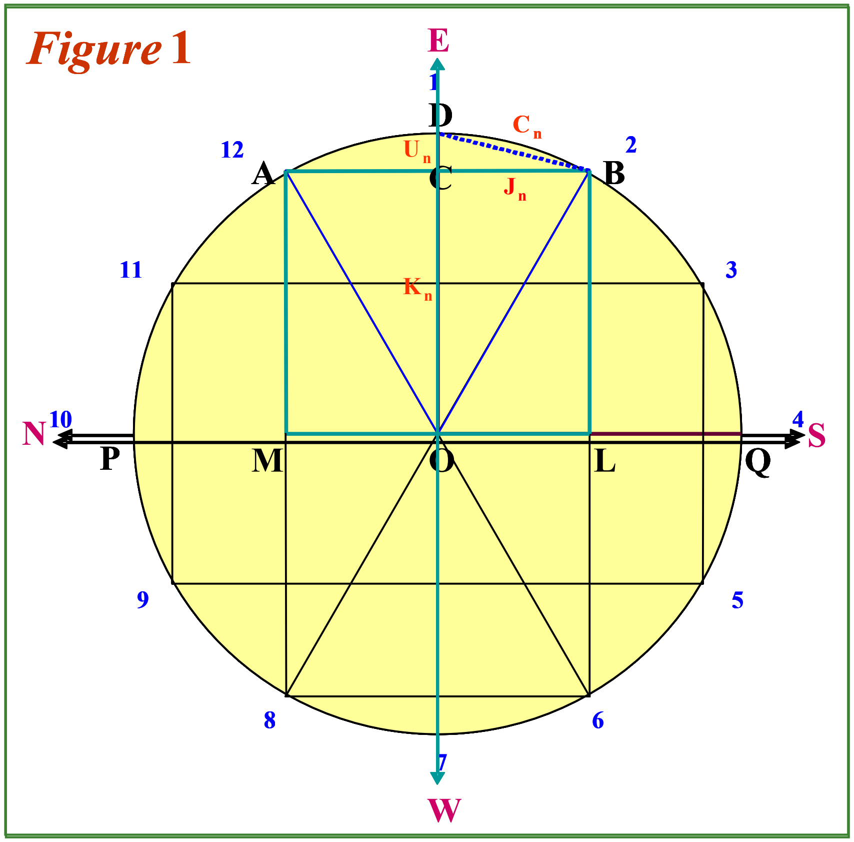
Draw vertical and horizontal lines passing through the centre of the circle to form two rectangles* (Figure 1), *dividing each quadrant into three equal arcs.

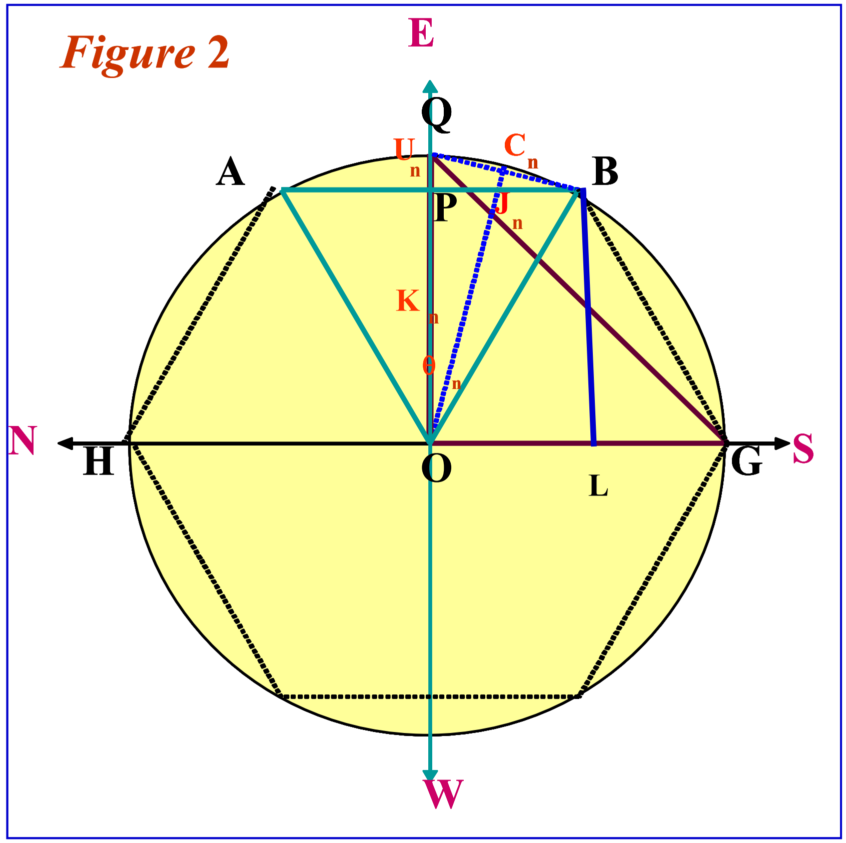
"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 77-83]

Figure to find D-rsines (contd) : -

***Aryabhatiya is a text on Astronomy, and therefore it has given instructions to draw* द्वादशभागा राशयः (*figure* 1).**

***Otherwise drawing a regular hexagon in a circle would be more convenient for Finding required number of* rsines*, as shown in figure* 2.**





"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 77-83]

To calculate 24-r*sines* from *Aryabhatiya-vyakya* by *Bhaskara I* (AD 629)

"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 77-83]

To calculate 16- r*sines*

पूर्वैवधालिखिते क्षेत्रे व्यासार्धमेव षोडशानां कष्ठानां [पूर्ण] ज्या । तदर्धमष्टानां काष्ठानां ज्या, साच १७१९ **(J_8= 1719)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्ग-विशेषस्य मूलं कोटिः(K_8=2977) । सा षोडशानां काष्ठानां ज्या, साच २९७७ **(J_{16}= 2977)** । एतां व्यासार्धाद्विशोधयेत् । शेषमष्टकाष्ठशरः (U_8=461) । शराष्टकाष्ठज्यावर्गयोगमूलं कर्णः । स एव अष्टानां कष्ठानां [पूर्ण] ज्या, साच २७८० । अर्धमस्याः चतुर्णां काष्ठानां ज्या, साच ८९० **(J_4= 890)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः (K_4=3321) । सैव विम्शतेः काष्ठानां ज्या, साच ३३२१ **(J_{20}= 3321)** । एतां व्यासार्धाद्वि-शोध्य शेषं चतुःकाष्ठशरः (U_4=117) । शरचतुष्काष्ठज्यावर्गयोगमूलं कर्णः । स एव चतुर्णां कष्ठानां [पूर्ण] ज्या, सा च ८९८ (C_4 = 898)। अर्धमस्याः काष्ठयोर्ज्या, सा च ४४९ **(J_2= 449)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः (K_2=3409) । सैव द्वाविंशतेः काष्ठानां ज्या, साच ३४०९ **(J_{22}= 3409)** । एतां व्यासार्धाद्विशोधयेत् । शेषं द्विकाष्ठशरः (U_2=29) । शरद्विकाष्ठज्यावर्गयोगमूलं कर्णः । स एव काष्ठयोः [पूर्ण] ज्या, सा च ४५०। अर्धमस्याः काष्ठस्य ज्या, सा च २२५ **(J_1= 225)** । एषा भुजा, व्यासर्धः कर्णः, भुजाकर्ण-वर्गविशेषस्य मूलं कोटिः (K_1=3431) । सैव त्रयोविम्शतेः काष्ठानां ज्या, साच ३४३१ **(J_{23}= 3431)** ।विशमत्वादतो ज्या नोत्पद्यते ।

"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 81-83]

To calculate 16- r*sines*

पूर्वैवधालिखिते क्षेत्रे व्यासार्धमेव षोडशानां कष्ठानां [पूर्ण] ज्या । तदर्धमष्टानां काष्ठानां ज्या, साच १७१९ **(J_8= 1719)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्ग-विशेषस्य मूलं कोटिः(K_8=2977) । सा षोडशानां काष्ठानां ज्या, साच २९७७ **(J_{16}= 2977)** । एतां व्यासार्धाद्विशोधयेत् । शेषमष्टकाष्ठशरः (U_8=461) । शराष्टकाष्ठज्यावर्गयोगमूलं कर्णः । स एव अष्टानां कष्ठानां [पूर्ण] ज्या, साच २७८० । अर्धमस्याः चतुर्णां काष्ठानां ज्या, साच ८९० **(J_4= 890)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः (K_4=3321) । सैव विम्शतेः काष्ठानां ज्या, साच ३३२१ **(J_{20}= 3321)** । एतां व्यासार्धाद्वि-शोध्य शेषं चतुःकाष्ठशरः (U_4=117) । शरचतुष्काष्ठज्यावर्गयोगमूलं कर्णः । स एव चतुर्णां कष्ठानां [पूर्ण] ज्या, सा च ८९८ (C_4 = 898)। अर्धमस्याः काष्ठयोर्ज्या, सा च ४४९ **(J_2= 449)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः (K_2=3409) । सैव द्वाविंशतेः काष्ठानां ज्या, साच ३४०९ **(J_{22}= 3409)** । एतां व्यासार्धाद्विशोधयेत् । शेषं द्विकाष्ठशरः (U_2=29) । शरद्विकाष्ठज्यावर्गयोगमूलं कर्णः । स एव काष्ठयोः [पूर्ण] ज्या, सा च ४५०। अर्धमस्याः काष्ठस्य ज्या, सा च २२५ **(J_1= 225)** । एषा भुजा, व्यासर्धः कर्णः, भुजाकर्ण-वर्गविशेषस्य मूलं कोटिः (K_1=3431) । सैव त्रयोविम्शतेः काष्ठानां ज्या, साच

३४३ *"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara"* – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 81-83]

To calculate 16- *rsines - contd*

अथ चतुर्णां काष्ठानां ज्यां द्विशोधयेत् । शेषं विंषतेः काष्ठानां शरः (U_{20}=2548)। शरविंशति-काष्ठज्यावर्गयोगमूलं कर्णः । स विंशतेः कष्ठानां [पूर्ण] ज्या, साच ४१८६। अर्धमस्याः दशानां काष्ठानां ज्या, साच २०९३ **(J_{10}= 2093)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः (K_{10}= 2727) । सैव चतुर्दशानां काष्ठानां ज्या, सा च २७२७ **(J_{14}= 2727)** । एतां व्यासार्धाद्विशोध्य शेषं दशकाष्ठानां शरः (U_{10}= 711) । शरदशकाष्ठज्यावर्गयोगमूलं कर्णः। स एव दशानां कष्ठानां [पूर्ण] ज्या, सा च २२१०। अर्धमस्याः पञ्चानां काष्ठानां ज्या, साच ११०५ **(J_5= 1105)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः । सैव एकोनविंशतेः काष्ठानां ज्या, सा च ३२५६ **(J_{19}= 3256)** । विशमत्वादतो ज्या नोत्पद्यते ।

अथ द्विकाष्ठज्यां व्यासार्धाद्विशोधयेत् । शेषं द्वाविंशतेः काष्ठानांशरः (U_{22}=2989) । शरद्वाविंशतिकाष्ठज्यावर्गयोगमूलं कर्णः। स एव द्वाविंशतेः कष्ठानां [पूर्ण] ज्या, सा च २५८५। अर्धमस्या एकदशानां काष्ठानां ज्या, साच १२९३ **(J_{11}= 1293)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः (K_{11}=3186) । सैव त्रयोदशनां काष्ठानां ज्या, सा च २५८५ **(J_{13}= 3186)** । विशमत्वादतो ज्या नोत्पद्यते ।

दशानां काष्ठानां ज्यां व्यासार्धाद्विशोधयेत् । शेषं चतुर्दशानां काष्ठानां शरः (U_{14}=711)।

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To calculate 16- r*sines - contd*

शरचतुर्दशकाष्ठज्यवर्गयोगमूलं कर्णः । स एव चतुर्दशानां कष्ठानां [पूर्ण] ज्या, साच २८१८। अर्धमस्याः सप्तानां काष्ठानां ज्या, साच १४०९ (J_7= 1409) । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः । सैव सप्तदशानां काष्ठानां ज्या, सा च ३१३६ (J_{17}=3136) । विशमत्वादतो ज्या नोत्पद्यते ।

To calculate 8- r*sines*

एवं त्रिभुजाद्राश्यष्टभागकाष्ठज्या व्याख्यातः। अथ चतुर्भुजाद् व्याख्यास्यामः। अन्तः समचतुरस्रस्यक्षेत्रस्य व्यासार्धतुल्या बाहवः । तयोर्वर्गयोगमूलः कर्णः । स एव चतुर्विंशतेः काष्ठानां [पूर्ण] ज्या, सा च ४८६२ (C_{24}= 4862) । अर्धमस्याः द्वादशानां काष्ठानां ज्या, साच २४३१ (J_{12}= 2431) | एतां व्यासार्धाद्विशोधयेत् । शेषं चतुर्दशानां काष्ठानां शरः। शरचतुर्दशकाष्ठज्यवर्गयोगमूलं कर्णः । स एव द्वादशानां काष्ठानां [पूर्ण] ज्या, साच २६३१। अर्धमस्याः षण्णां काष्ठानां ज्या, साच १३१६ (J_6= 1316) । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः । सा अष्टादशानां काष्ठानां ज्या, सा च ३१७७ (J_{18}= 3177) । एतां व्यासार्धाद्विशोधयेत् । शेषं षण्णां काष्ठानां शरः। शरषट्काष्ठज्यवर्गयोगमूलं कर्णः । स एव षण्णां कष्ठानां [पूर्ण] ज्या,

"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p81-83]

To calculate 8- r*sines - contd*

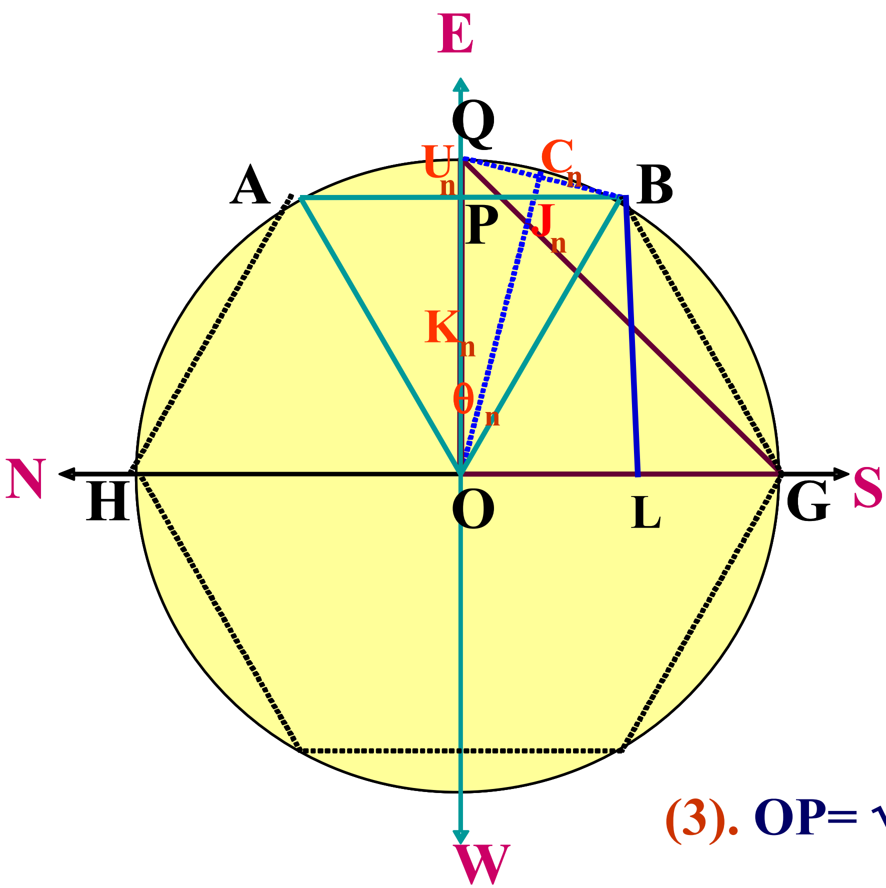
साच १३४२। अर्धमस्याः त्रयाणां काष्ठानां ज्या, साच ६७१ **(J_3= 671)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः । सैव एकविंशतेः काष्ठानां ज्या, सा च ३३७२ **(J_{21}= 3372)** । विशमत्वादतो ज्या नोत्पद्यते ।

अथ षण्णां काष्ठानां ज्यां व्यासार्धाद्विशोधयेत् । शेषं अष्टादशकाष्ठानां शरः। शराष्टादशकाष्ठज्यावर्गयोगमूलं कर्णः । स एव अष्टादशानां कष्ठानां [पूर्ण] ज्या, साच ३८२०। अर्धमस्या नवानां काष्ठानां ज्या, साच १९१० **(J_9=1910)** । एषा भुजा, व्यासार्ध कर्णः, भुजाकर्णवर्गविशेषस्य मूलं कोटिः । सैव पञ्चदशानां काष्ठानां ज्या, सा च २८५९ **(J_{15}= 2859)** । विशमत्वादतो ज्या नोत्पद्यते ।

एवं राश्यष्टभागकाष्ठज्याश्चतुर्विंशतिः । अनेनैव विधानेन विश्कम्भार्धे यथेष्टानि-ज्यार्धानि निष्पादयितव्यानि इति ॥ ११ ॥

"Aryabhatiya of Aryabhata with the commentary of Bhaskara-I and Somesvara" – critically edited by K S Shukla, The Indian National Science Academy, Bahadur Shaw Zafar Marg, New Delhi – 1 (1976) [Ref.1, p 81-83]

To calculate D-r*sines* from *Aryabhatiya* (AD 499) Algorithm



Divide arc QBG of quadrant QBGO into D equal arcs, where $D = 6x(2)^{(t-1)}$

Arc QB = arc of n equal arcs.

$\angle QOB = \theta_n$,

$PB = (1/2)\ AB = r\ \text{sine}\theta_n$

$= \text{Jya of n arcs} = J_n$

(1). $AB = C_{2n}$ = Chord of 2n arcs.

where $2n = \dfrac{2D}{3}$

(2). $J_n = (1/2)\ C_{2n}$

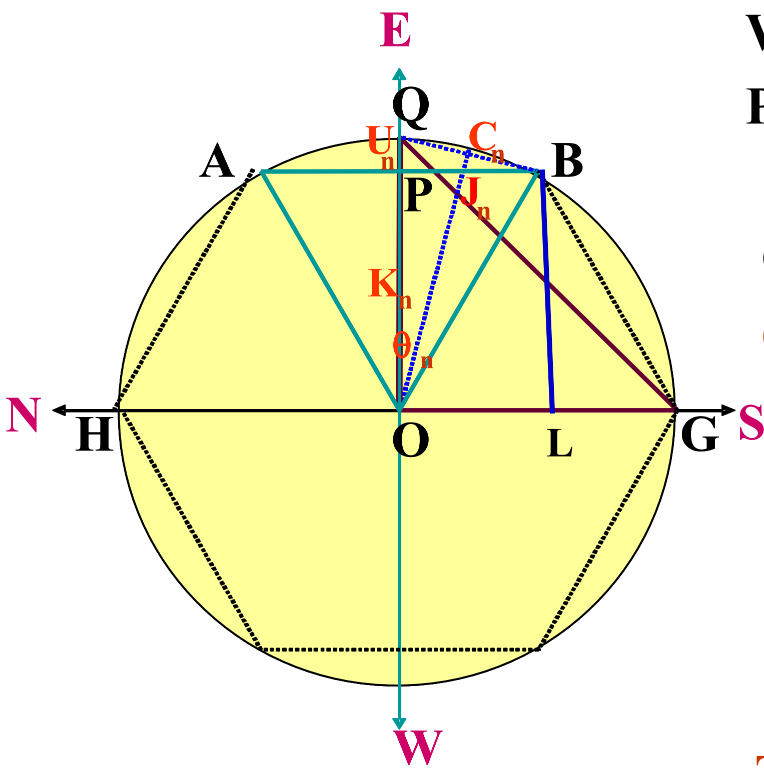
(3). $OP = \sqrt{(r^2 - J_n^2)}$ = kotijya of n arcs $= K_n = r \cos \theta_n$

[Arc BG = arc of (D-n) equal arcs]

$OP = LB = r \sin (\angle BOL) = r \sin (90 - \theta_n) = J_{D-n}$

(4). $K_n = J_{D-n}$

To calculate D-r*sines* from *Aryabhatiya* (AD 499) Algorithm



When n or (D - n) = 2m, an even number

PQ = $(r - K_{2m})$ = U_{2m} = Utkrama Jya of 2m arcs

(5). $U_{2m} = r - K_{2m}$

QB = $\sqrt{(J_{2m}^2 + U_{2m}^2)}$ = chord of 2m arcs = C_{2m}

(6). $C_{2m} = \sqrt{(J_{2m}^2 + U_{2m}^2)}$ = chord of 2m arcs

(7). (6) is similar to (1).

Repeat steps from (2) to (6)

To find D-*rsines*: -

I. Find 2m-*rsines* [where 2m =(2D/ 3)] assuming chord AB as a side of the inscribed hexagon with 3438 units as radius having 2m equal arcs = C_{2m} = 3438.

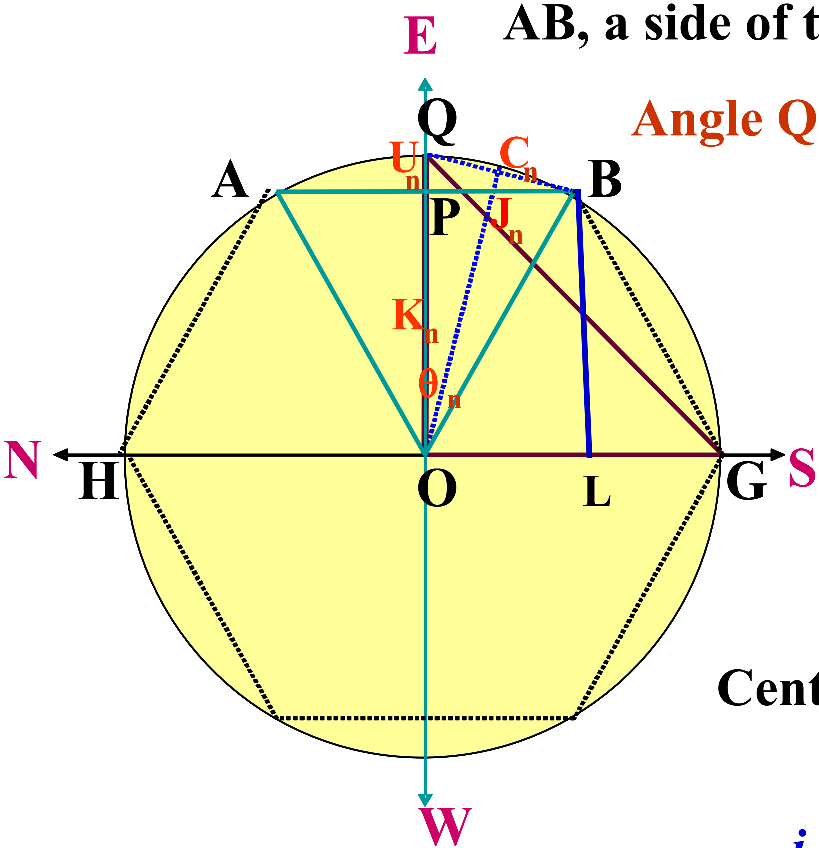
II. Find m-*rsines* [where m =(D/ 3)] assuming Chord QG as the hypotenuse of an isosceles right triangle QOG with 3438 as a chord of D arcs = C_D = $3438\sqrt{2}$ = 4862

To find 24-rsines on the basis of an illustrative example.

Draw an inscribed hexagon having the radius 3438 *units* (one radian measure in *minutes*).

AB, a side of the hexagon = radius of the circle = 3438 *units*.

Angle QOG = Central angle made by arc QBG = 90°.



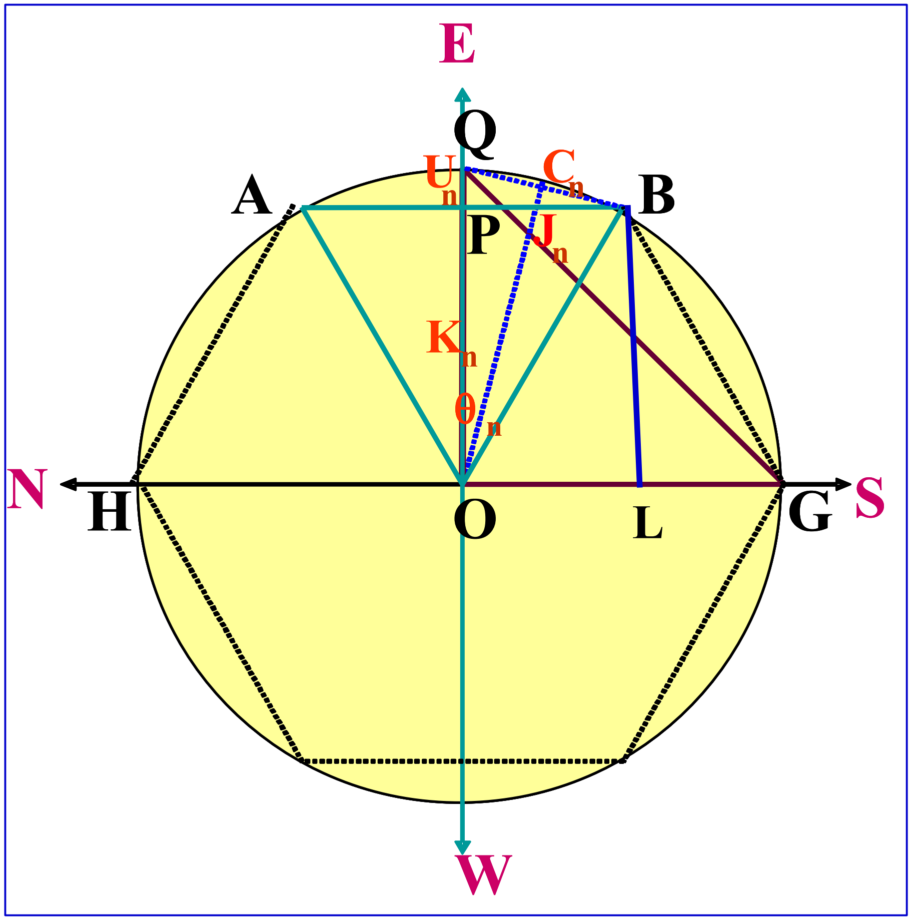
Arc QBG is divided into 24 (= D) equal arcs.

Central angle made by chord AB [*radius of inscribed hexagon*] = angle AOB = 60°.

Central angle made by chord PB = $\frac{90^0 n}{24}\left(=\frac{90^0 n}{D}\right)$

***i.e.*, by the chord of n equal arcs at centre O.**

A general method to find D-r*sines based on*
an illustrative example for finding 24-r*sines* .



To find 24-*rsines* (D-*rsines*): -

I. Find 16-r*sines*

[where 2m =(2D/ 3) = 16]
assuming Chord AB as a side of the inscribed hexagon with 3438 units as radius having 16 equal arcs = C_{16} = 3438 (= C_{2m}).

II. Find 8-r*sines*

[where m =(D/ 3) = 8]
assuming Chord QG as the hypotenuse of isosceles right triangle QOG as a Chord of 24 arcs = C_{24} = 4862 (= C_D) = 3438√2]

Next two tables based on the explanation in Aryabhatiya-bhasya of Bhaskara I, gives a comparison of Aryabhatiya and present-day values of 24-rsines

To calculate 16- r*sines*

C_{2n}	J_n K_{D-n}	K_n	U_n	J_n /3438	θ = [90n /24]	Sine θ
	J_{16}= 2977			0.8659	60	0.8660
C_{16} =3438	J_8= 1719	K_8= 2977	U_8= 461	0.5000	30	0.5000
C_8 = 1780	J_4= 890	K_4= 3321	U_4= 117	0.2589	15	0.2588
	J_{20}= 3321			0.9660	75	0.9660
C_4 = 898	J_2= 449	K_2= 3409	U_2= 29	0.1306	7.5	0.1305
	J_{22}= 3409	K_{22}= 446	U_{22}= 2992	0.9916	82.5	0.9914
C_2 = 450	J_1 = 225	K_1= 3431		0.0654	3.75	0.0654
	J_{23}= 3431			0.9980	86.25	0.9979
C_{22} = 4536	J_{11}= 2268	K_{11}= 2584		0.6596	41.25	0.6593
	J_{13}= 2584			0.7516	48.75	0.7518
		K_{20}= 890	U_{20}= 2548			
C_{20} = 4186	J_{10}= 2093	K_{10}= 2727	U_{10}= 711	0.6088	37.5	0.6088
	J_{14}= 2727			0.7931	52.5	0.7934
C_{10}= 2290	J_5 = 1105	K_5= 3256		0.3214	18.75	0.3214
	J_{19} = 3256			0.9470	71.25	0.9469
		K_{14}= 2094	U_{14}= 1344			
C_{14}= 3040	J_7 = 1520	K_7 = 3084		0.4421	26.25	0.4423
	J_{17} = 3084			0.8970	63.75	0.8969

To calculate 8 -r*sines*

C_{2n}	J_n K_{D-n}	K_n	U_n	J_n /3438	θ = [90n /24]	Sine θ
C_{24}= 4862	J_{12}= 2431	K_{12}= 2431	U_{12}= 1007	0.7071	45	0.7071
C_{12} =2631	J_6= 1316	K_6= 3176	U_6= 262	0.3828	22.5	0.3827
C_6 = 1342	J_3 = 671	K_3= 3372		0.1952	11.25	0.1951
	J_{21} = 3372	K_{21}= 670		0.9808	78.75	0.9808
	J_{18} = 3176	K_{18}= 1316	U_{18}= 2122	0.9238	67.5	0.9239
C_{18}= 3820	J_9 = 1910	K_9= 2859		0.5556	33.75	0.5556
	J_{15}= 2859			0.8316	56.25	0.8315
	J_{24}= 3438			1.0000	90	1.0000

Reason for considering 24-r*sines* - *Suryadeva yajvan* (b. A.D. 1191)

“चतुर्विंशतिधा चापखण्डने कृते प्रथमज्यार्धं चापं च तुल्यसङ्ख्यं जातम्।”

‘On dividing a quadrant into 24 equal parts,
***the first* rsine *and the corresponding arc are same’*.**

***Jya of* 1 *arc* = J_1 =225 = 3438 x sin 3.75^0 = 3438 x 0.0654 = 224. 86 = 225 (app.),**
where* [(90x1)/ 24] = 3.75^0 = 225 *minutes.

This is almost equivalent to sin θ = θ, when θ is very small.

Ref.: *Aryabhatiya, with the commentary of Suryadeva Yajvan* : Edited by K V Sharma, INSA, New Delhi, (1976) p. 47]

Explanation for taking 3438 as the circum-radius
- *Suryadeva yajvan* (b. A.D. 1191)

Suryadeva yajvan has explained the reason for taking 3438 as the circum-radius for all scientific calculations.

यदि *रदवसुयमलरस* (62,832) मितपरिधेर् *अयुतद्वय* (20,000) व्यासः *खखषड्घन* (21,600) लिप्तात्मक मितपरिधेश्चक्रस्य को व्यासः इति।

'If 62,832 is the perimeter of a circle of diameter 20.000 units, What is the circum-diameter of a circle of perimeter 21,600 units?'

_अयुतद्वय_ (20,000) _चक्रकला_ (21600)-भिर्हत्वा _रदवसुयमलरस_ (62,832)-स्य विभज्य लब्धं _चक्रव्यासः_। तदर्धं चक्रव्यासार्धं _वस्वाग्निवेदराम_ (3438) सङ्ख्यम्। अनेन व्यासार्धेन शास्त्रीयस्सकलो व्यवहारः।

'When 20.000 *multiplied by* (*the unit equivalent to*) *central angle made by the perimeter in minutes* (21,600) and *the product divided by the perimeter* (62,832) gives *the circum-diameter'*.

Ref.: *Aryabhatiya, with the commentary of Suryadeva Yajvan* : Edited by K V Sharma, INSA, New Delhi, (1976) p. 47]

Explanation for taking **3438** *as the circum-radius*:
by *Suryadeva yajvan* (b. A.D. 1191)

'When 20.000 ***multiplied by*** **(*****the unit equivalent to*****)** ***central angle made by the perimeter in minutes*** **(21,600) and** ***the product divided by the perimeter*** **(62,832) gives** ***the circum-diameter'*****.**

$$\textit{circum-diameter} = \frac{20,000 \times 21,600}{62,832} = 6876\left(app.\right)$$

Half of **6876 = 3438 units** ***is used for all scientific calculations.***

Ref.: *Aryabhatiya, with the commentary of Suryadeva Yajvan* : Edited by K V Sharma, INSA, New Delhi, (1976) p. 47]

Present-day Radian measure (in minutes) and from Aryabhatiya

The radian is the central angle made by an arc of length equal to the radius of the circle.

It requires the ratio of perimeter of the circle to its diameter.

The ratio of perimeter of a circle to its diameter is named π, because in Greek language 'perimeter' is 'περιμετερ' and π is its first alphabet.

$$One\ radian = \frac{180 \times 60}{\pi} = 3437.746771 = 3438\ \left(app.\right)$$

Aryabhata-I used a circle of radius* 3438 *units to find* D-r*sines.

3438 ***is the measure of 'one radian' in minutes approximately.***

Perimeter of a Circle of diameter 20,000 *units in Aryabhatiya*

Aryabhatiya **of Aryabhata-I has given**

the circumference of a circle of diameter 20,000 units in a s*loka*;

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।

अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥१०॥ [Refer; 1. (p.71)]

'Four more than hundred multiplied by eight and increased by sixty-two thousand is a nearer value* (आसन्न) *of the perimeter of a circle of diameter twenty-thousand units'.

[(100 + 4) 8 + 62,000] = 62,832

is a nearer value of the Perimeter of a circle of diameter **20,000** ***units.***

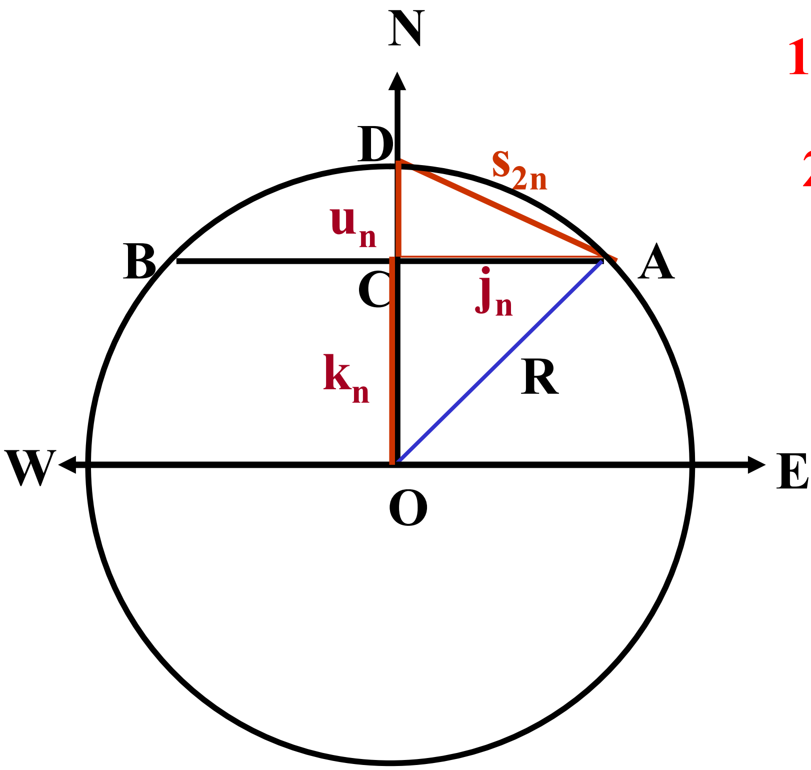
The ratio of perimeter **62,832** ***units to its diameter*** **20,000** ***units is the Aryabhatiya value of*** **π;**

$$\pi = \frac{62,832}{20,000} = 3.1416$$

[Refer: ***Aryabhatiya, with the commentary of Bhaskara-I and Someswara*** : Edited by K S Shukla, INSA, New Delhi, (1976) p.71]

Discussion: -

Algorithm to *double the number of sides* 'n' of an Inscribed Regular Polygon



1) AB = a side of n-sided regular polygon = s_n

2) p_n = Perimeter of the polygon = nAB = ns_n

3) AC = (1 / 2) AB = (1/2) s_n = j_n

4) OC= $\sqrt{(OA^2 - AC^2)} = \sqrt{(r^2 - j_n^2)} = k_n$

5) CD = (OD – OC) = (r - k_n) = u_n

6) AD = $\sqrt{(AC^2 + CD^2)} = \sqrt{(j_n^2 + u_n^2)} = s_{2n}$

s_{2n} = side of 2n-sided regular polygon.

Steps 1 to 6 (*except step* 2) *converts* a side of n-sided regular polygon to a side of 2n-sided regular polygon. Step 2 gives perimeter of inscribed regular polygon of n sides, $p_n = ns_n$

The following table shows the *Aaryabhata's* approximation corresponding to the regular 6 x 2^{t-1}–sided regular polygon, when t = 1;

i.e., of a regular hexagon inscribed in a circle of diameter 20,000 units.

To find circumference of a circle of diameter 20,000 units in Excel Programer

t	n = 6*2^(t-1)	S_n	P_n=n*S_n	J_n=(S_n/2)	K_n=sqrt(1000^2-J_n^2)	U_n=(1000-K_n)	S_{2n}=sqrt(J_n^2+U_n^2)	(P_n/20000)
1	6	10000	60000	5000	8660.254038	1339.745962	5176.380902	3
2	12	5176.380902	62116.57082	2588.19045	9659.258263	340.7417371	2610.523844	3.105828541
3	24	2610.523844	62652.57227	1305.26192	9914.448614	85.55138626	1308.062585	3.132628613
4	48	1308.062585	62787.00406	654.031292	9978.589232	21.41076761	654.3816564	3.139350203
5	96	654.3816564	62820.63902	327.190828	9994.645875	5.354125236	327.2346325	3.141557608
6	192	327.2346325	62829.04945	163.617316	9998.661379	1.338620904	163.6227921	3.141452472
7	384	163.6227921	62831.15216	81.811396	9999.665339	0.334660826	81.81208052	3.141557608
8	**768**	81.81208052	**62831.67784**	40.9060403	9999.916334	0.083665556	40.90612582	**3.141583892**
9	1536	40.90612582	62831.80926	20.4530629	9999.979084	0.020916411	20.45307361	3.141590463
10	3072	20.45307361	62831.84212	10.2265368	9999.994771	0.005229104	10.22653814	3.141592619
11	6144	10.22653814	62831.85033	5.11326907	9999.998693	0.001307276	5.113269237	3.141592517
12	12288	5.113269237	62831.85239	2.55663462	9999.999673	0.000326819	2.55663464	3.141592619
13	24576	2.55663464	62831.8529	1.27831732	9999.999918	8.17048E-05	1.278317322	3.141592645
14	49152	1.278317322	62831.85303	0.63915866	9999.99998	2.04262E-05	0.639158662	3.141592651
15	98304	0.639158662	62831.85306	0.31957933	9999.999995	5.10655E-06	0.319579331	3.141592654
16	196608	0.319579331	62831.85307	0.15978967	9999.999999	1.27664E-06	0.159789665	3.141592653
17	393216	0.159789665	62831.85307	0.07989483	10000	3.1916E-07	0.079894833	3.141592654
18	786432	0.079894833	62831.85307	0.03994742	10000	7.979E-08	0.039947416	3.141592654
19	1572864	0.039947416	62831.85307	0.01997371	10000	1.9947E-08	0.019973708	3.141592654
20	3145728	0.019973708	62831.85307	0.00998685	10000	4.98585E-09	0.009986854	3.141592654
21	6291456	0.009986854	62831.85307	0.00499343	10000	1.24601E-09	0.004993427	3.141592654
22	12582912	0.004993427	62831.85307	0.00249671	10000	3.11047E-10	0.002496714	3.141592654
23	25165824	0.002496714	62831.85307	0.00124836	10000	7.82165E-11	0.001248357	3.141592654
24	50331648	0.001248357	62831.85307	0.00062418	10000	2.00089E-11	0.000624178	3.141592654
25	100663296	0.000624178	62831.85307	0.00031209	10000	0	0.000312089	3.141592654
26	**201326592**	0.000312089	**62831.85307**	0.00015604	10000	0	0.000156045	**3.141592654**

$$\pi = \frac{62831.67784}{20000} = 3.141583892 = 3.141$$

(Ganesha)

$$\pi = \frac{62831.85307}{20000} = 3.141592654 = 3.1416$$

To find circumference of a circle of diameter 20,000 units in Excel Programer

t	n = 6*2^(t-1)	Sn	Pn=n*Sn	Jn=(Sn/2)	Kn=sqrt(1000^2-Jn^2)	Un=(1000-Kn)	S2n=sqrt(Jn^2+Un^2)	(Pn/20000)
1	6	10000	60000	5000	8660.254038	1339.745962	5176.380902	3
2	12	5176.380902	62116.57082	2588.19045	9659.258263	340.7417371	2610.523844	3.105828541
3	24	2610.523844	62652.57227	1305.26192	9914.448614	85.55138626	1308.062585	3.132628613
4	48	1308.062585	62787.00406	654.031292	9978.589232	21.41076761	654.3816564	3.139350203
5	96	654.3816564	62820.63902	327.190828	9994.645875	5.354125236	327.2346325	3.141557608
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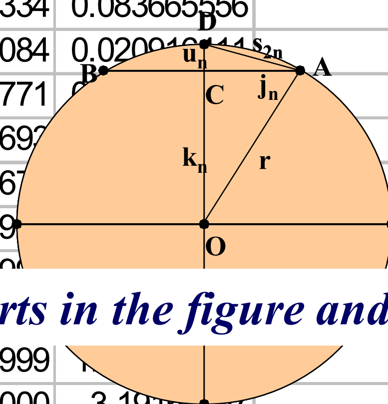
7	384	163.6227921	62831.15216	81.811396	9999.665339	0.334660826	81.81208052	3.141557608
8	**768**	81.81208052	**62831.67784**	40.9060403	9999.916334	0.083665556	40.90612582	**3.141583892**
9	1536	40.90612582	62831.80926	20.4530629	9999.979084	0.0209[illegible]	20.45307361	3.141590463
10	3072	20.45307361	62831.84212	10.2265368	9999.994771	[illegible]	10.22653814	3.141592619
11	6144	10.22653814	62831.85033	5.11326907	9999.99869[illegible]	[illegible]	5.113269237	3.141592517
12	12288	5.113269237	62831.85239	2.55663462	9999.9996[illegible]	[illegible]	2.55663464	3.141592619
13	24576	2.55663464	62831.8529	1.27831732	9999.9999[illegible]	[illegible]	1.278317322	3.141592645
14	49152	1.278317322	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]592651
15	98304	0.639158662	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]592654
16	196608	0.319579331	62831.85307	0.15978967	9999.999999	[illegible]	0.159789665	3.141592653
17	393216	0.159789665	62831.85307	0.07989483	10000	3.19[illegible]	0.079894833	3.141592654
18	786432	0.079894833	62831.85307	0.03994742	10000	7.979E-08	0.039947416	3.141592654
19	1572864	0.039947416	62831.85307	0.01997371	10000	1.9947E-08	0.019973708	3.141592654
20	3145728	0.019973708	62831.85307	0.00998685	10000	4.98585E-09	0.009986854	3.141592654
21	6291456	0.009986854	62831.85307	0.00499343	10000	1.24601E-09	0.004993427	3.141592654
22	12582912	0.004993427	62831.85307	0.00249671	10000	3.11047E-10	0.002496714	3.141592654
23	25165824	0.002496714	62831.85307	0.00124836	10000	7.82165E-11	0.001248357	3.141592654
24	50331648	0.001248357	62831.85307	0.00062418	10000	2.00089E-11	0.000624178	3.141592654
25	100663296	0.000624178	62831.85307	0.00031209	10000	0	0.000312089	3.141592654
26	**201326592**	0.000312089	**62831.85307**	0.00015604	10000	0	0.000156045	**3.141592654**



Tally the lengths of different parts in the figure and in the table.

$$\pi = \frac{62831.67784}{20000} = 3.141583892 = 3.141($$

(Ganesha)

$$\pi = \frac{62831.85307}{20000} = 3.141592654 = 3.1416$$

Value of π (*to* 1000 *digits*)

π = 3.141592653 58979323846264338327950288419716939937510582097
494459230781640628620899862803482534211706798214808651328230
664709384460955058223172535940812848111745028410270193852110
555964462294895493038196442881097566593344612847564823378678
316527120190914564856692346034861045432664821339360726024914
127372458700660631558817488152092096282925409171536436789259
036001133053054882046652138414695194151160943305727036575959
195309218611738193261179310511854807446237996274956735188575
272489122793818301194912983367336244065664308602139494639522
473719070217986094370277053921717629317675238467481846766940
513200056812714526356082778577134275778960917363717872146844
090122495343014654958537105079227968925892354201995611212902
196086403441815981362977477130996051870721134999999837297804
995105973173281609631859502445945534690830264252230825334468
503526193118817101000313783875288658753320838142061717766914
730359825349042875546873115956286388235378759375195778185778
053217122680661300192787661119590921642019 9....

Perimeter of an *inscribed regular polygon of sides* **20,13,26,592** *in a circle of diameter* **20000** *units* = **62831.85307** *- by Aryabhatiya Algorithm*

$$\pi = \frac{62831.85307}{20000} = 3.141592654$$

Value of π *from Lilavati of Bhaskara –II* (born 1114. A.D.)

व्यासे भनन्दाग्नि हते विभक्ते खबाणसूर्यैः परिधिस्ससूक्ष्मः ।

भनन्दाग्नि =3927	खबाणसूर्यैः = 1250
भ = 27; नन्दः = 9; अग्निः = 3	ख = 0; बाणः = 5; सूर्यः = 12

अङ्कानाम् वामतोगतिः ।

$$\textit{Circumference} = \frac{3927\text{x } diameter}{1250} = 3.1416\, d = \pi\, d$$ (सूक्ष्मः= ***minute value***)

Minute Value of $\pi = 3.1416$, *suitable for scientific calculations.*

द्वाविंशतिघ्ने विहृतेथ शैलैः स्थूलोथवा स्याद्व्यवहारयोग्यः ॥२०७॥

$$\textit{Circumference} = \frac{22\text{ x } diameter}{7} = 3.1429\, d = \pi\, d$$ (व्यवहारयोग्यः= ***suitable for daily use***)

General Value of $\pi = 3.1429$, *suitable for daily usage.*

Lilavati of Bhakaracarya, A Treatise of Vedic Tradition ; Translated by K S Patwardhan, S A Naimpally, S L Singh, Motilal Banarsidass Publishers Private Limited, Delhi (2001) p143

Power Series of π of *Madhava* (*c.* 1350 - 1410) and *Gottfried Wilhelm Leibnitz* (c. 1646 - 1716)

Madhava (*c.* 1350 - 1410) of *Sangamagrama* anticipated the power series of π attributed to *Gottfried Wilhelm Leibnitz* (1646 - 1716) in the sloka that gives series to get the circumference (C) of a circle;

व्यासे वारिधि निहते रूपहृते व्यससागराभिहते।
त्रि-शरादि-विशमसङ्ख्या भक्तं ऋणां पृथक् क्रमात् कुर्यात्॥

"Multiply the diameter by four. Subtract from it and add to it alternately the quotients obtained *while dividing four times the diameter* by the odd integers three, five and so on.
(*to get the circumference of the circle*)".

Therefore, $\pi d = 4d - \dfrac{4d}{3} + \dfrac{4d}{5} - + \ . \ . \ . = C$ ***- Madhava Series***

Dividing by 4d ; $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - + \ . \ . \ .$ ***- Leibnitz Series***

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Thank you