PLACE-VALUE SYSTEM IN NUMERALS

A problem

Ask your friend to assume any three-digit number **P** in which the first and the last digits differ by *two* or *more than two*.

537

735

Interchange the *digits in the place-values hundred and unit place of* the number P to form a new number Q.

	100	10	1
P	7	3	5
Q	5	3	7
P – Q = R	1	9	8

Find (P - Q) or (Q - P), a positive difference R

Interchange the *digits in the place-values hundred and unit place of* the number R to form a new number S.

	100	10	1
R	1	9	8
S	8	9	1
R + S = T	10	8	9

Find the sum of R and S; R + S = T

Final answer = 1089

Observing the pattern

#	Step 1	Step 2	Step 3	Step 4	Step 5
	A	B	|(A-B)| = C	D	(C + D) = E
1	376	673	(673 – 376) = 297	792	(297 + 792) = 1089
2	508	805	(805 – 508) = 297	792	(297 + 792) = 1089
3	790	097	(790 – 097) = 693	396	(693 + 396) = 1089
4	239	932	(932 – 239) = 693	396	(693 + 396) = 1089
5	139	931	(931 – 139) = 792	297	(792 + 297) = 1089
6	429	924	(924 – 429) = 495	594	(495 + 594) = 1089
7	204	402	(402 – 204) = 198	891	(198 + 891) = 1089

Observe the examples given in the table.

Can you generalize the above process?

Explain the reason for this surprising result.

Justifying the problem

Assume any three-digit number A : $100a + 10b + c$ in which the first and the last digits differ by *two* or *more than two*.

Interchange the *digits in the place-values hundred and unit place of* the number A to form a new number B : $100c + 10b + a$.

Find (A – B) or (B – A), a positive difference C. Assume A > B

A : $100a + 10b + c$

B : $100c + 10b + a$

A – B = C : $(a - c)100 + 0 + (c - a)$

C could be made to have 9 x 10 (= 90) by subtracting and adding 100

C : $(a - c - 1)100 + 9 \times 10 + (10 + c - a)$

D : $(10 + c - a)100 + 9 \times 10 + (a - c - 1)$

Interchange the *digits in the place-values hundred and unit place of* the number C to form a new number D.

C + D : $(-1 + 10)\,100 + 18 \times 10 + (10 - 1)$

$= 900 + 180 + 9$

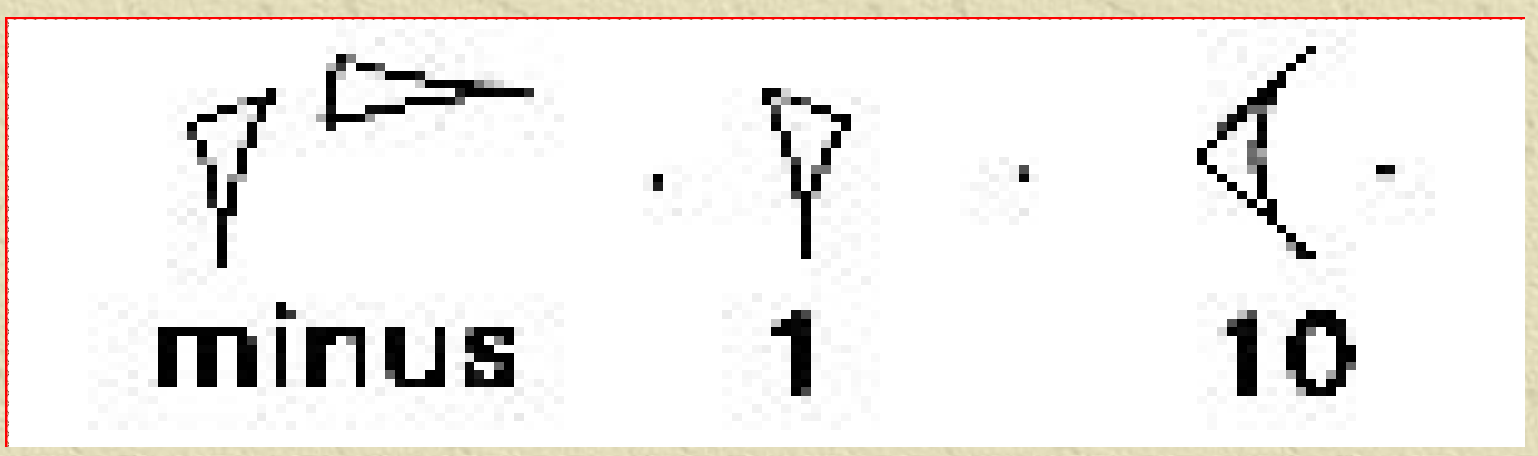
$= 1089$

Find the sum of C and D

The Place-value Principle for Numerals - Independently conceived Only Four times

- By the Indians; before 2500 BC.
- By the Babylonians; probably in the early 2nd millennium BC.
- By the Mayas; probably during 3rd to 9th century AD.
- By the Chinese; shortly before the beginning of the Christian era.

Babylonian Digits with a sign for minus



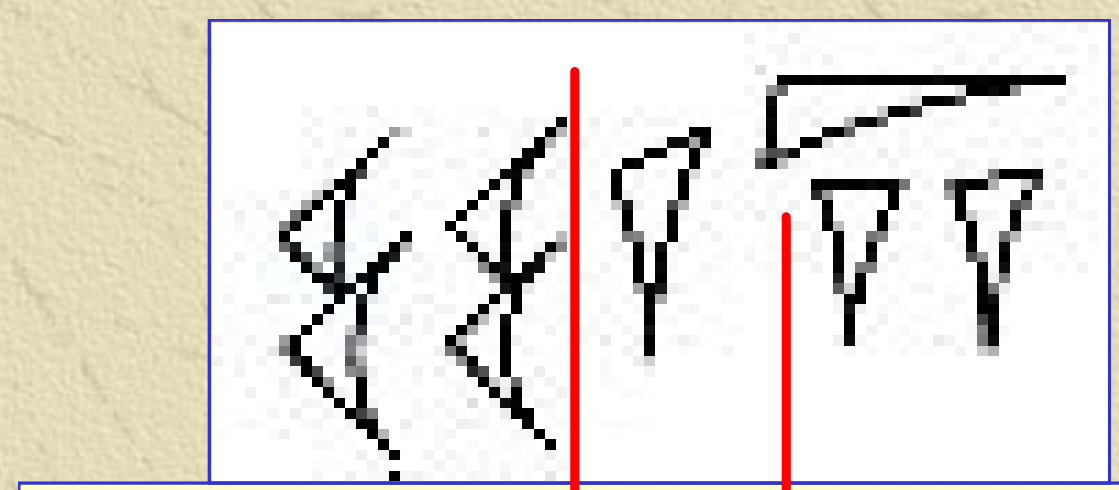
Importance of *Ganita* in Indian Sciences (Vedas)

यथा शिखा मयूराणं नागानां मणयो यथा ।
तद्वद् वेदाङ्ग शास्त्राणां गणितं मूर्धनि स्थितम् ॥

Like the combs of the peacocks and
the crest jewels of the serpents,
so does the tradition of mathematics knowledge stand
at the head of all the tradition of Sciences knowledge
forming the auxiliaries of the Vedas.

[Refer; *Vedanga jyotisa of Lagadha* – the translation and notes of Prof. T.S. Kuppanna Sastry, edited by K.V. Sarma, Indian National Science Academy,New Delhi (p.27 & 36) 1985]

Babylonian Numeral for *thirty-eight*



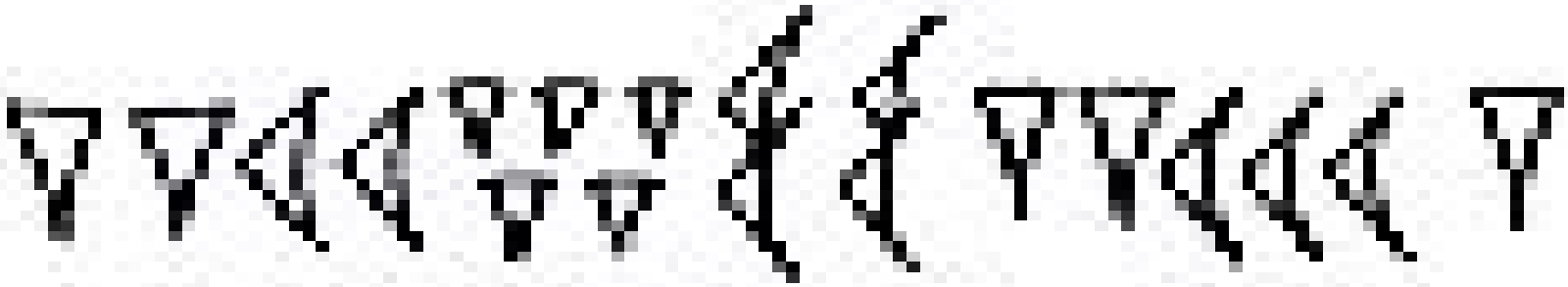
38 = 40 - 2

Babylonian Numerals for numbers from one to fifty-nine

1 	11 	21 	31 	41 	51 
2 	12 	22 	32 	42 	52 
3 	13 	23 	33 	43 	53 
4 	14 	24 	34 	44 	54 
5 	15 	25 	35 	45 	55 
6 	16 	26 	36 	46 	56 
7 	17 	27 	37 	47	57
8	18	28	38	48	58
9	19	29	39	49	59
10	20	30	40	50	

Babylonian Numerals for numbers Greater than Fifty-nine.

The Babylonian numeral **for a number 5,24,551 in cuneiform is represented as;**



$= 2(60^3) + 25(60^2) + 42(60) + 31$

$= 5,24,551$

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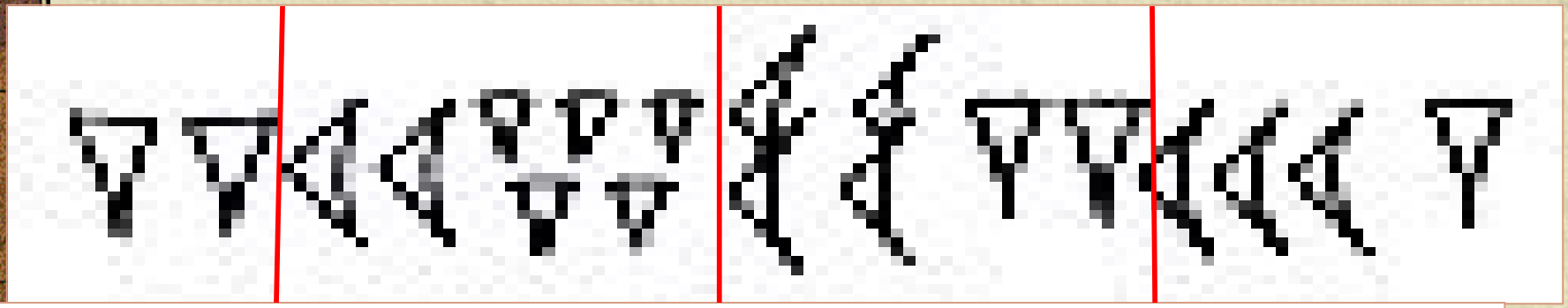
The Babylonian numeral **for a number 5,24,551 in cuneiform is represented as;**

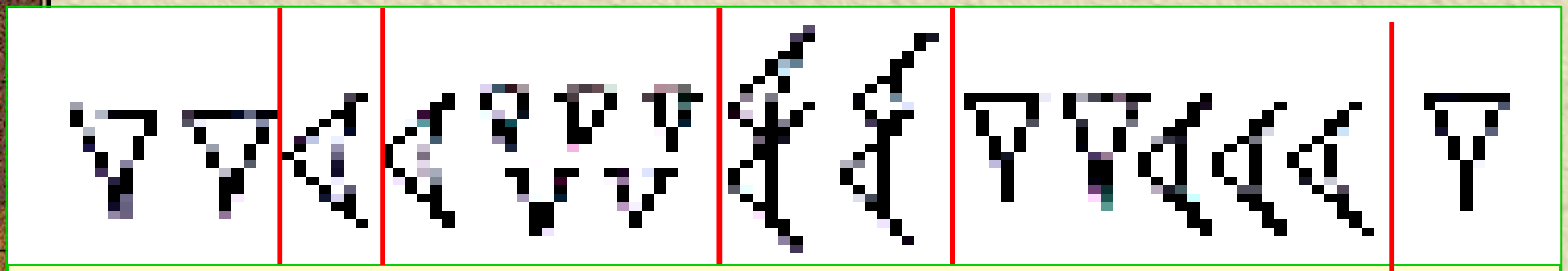


$= 2(60^3) + 25(60^2) + 42(60) + 31$

$= 5,24,551$

Difficulties of the Babylonian System of Numerals

The cuneiform numeral for 524551, which was in sexagesimal system (*base* 60) could also be read as



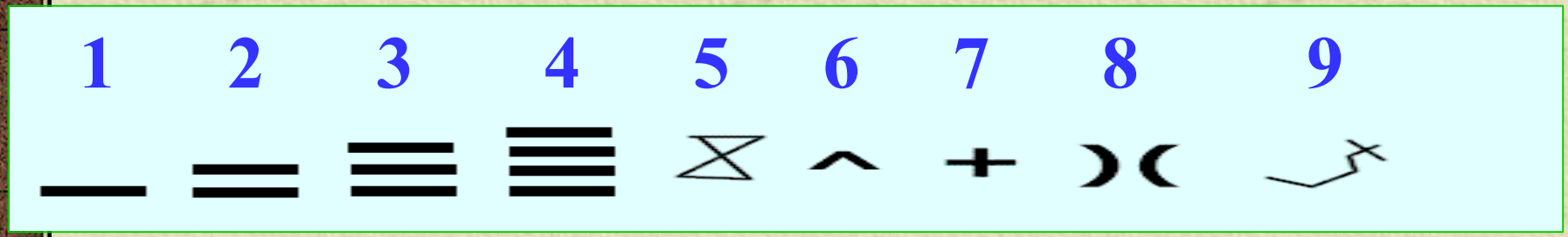
$$=2(60^5)+10(60^4)+15(60^3)+40(60^2)+23(60^1)+1$$

$$= 1,68,81,85,381$$

Another difficulty in the Babylonian system was it had no zero until about 300 B.C.

Chinese Numerals

The Chinese numerals **for the numbers one to nine :**



China **has adopted the**
Hindu-Arabic numeral system of base ten.

It also uses its native Chinese character number system.

The Chinese system **has important differences in the way the numbers are represented.**

A Brief Description of the Chinese Number System

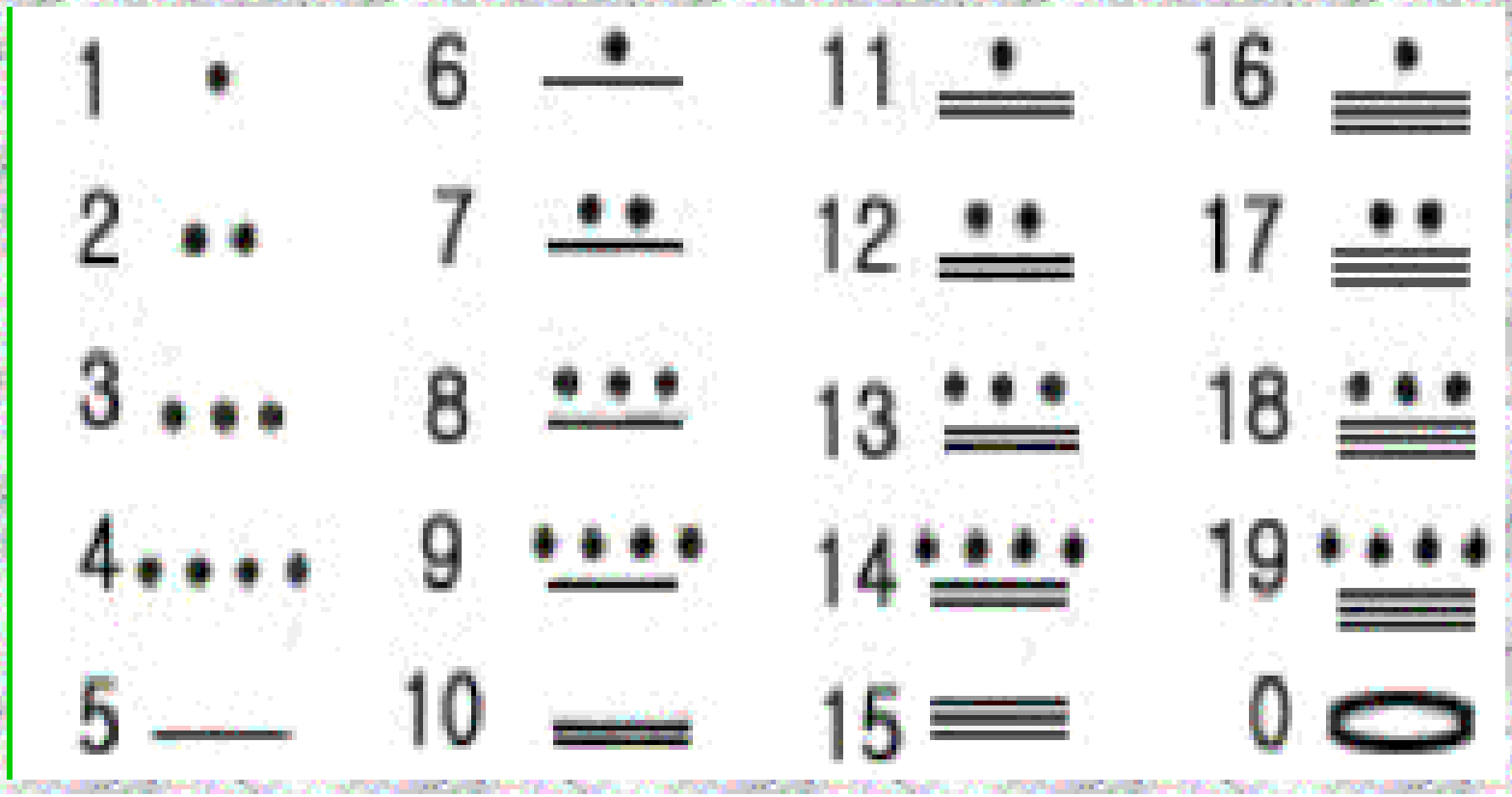
		零	0				
一	1	五	5	九	9		
二	2	六	6	十	10		
三	3	七	7	百	100		
四	4	八	8	千	1000		
萬	10000			億	100000000		

Chinese Symbol for Zero

A smiling Face

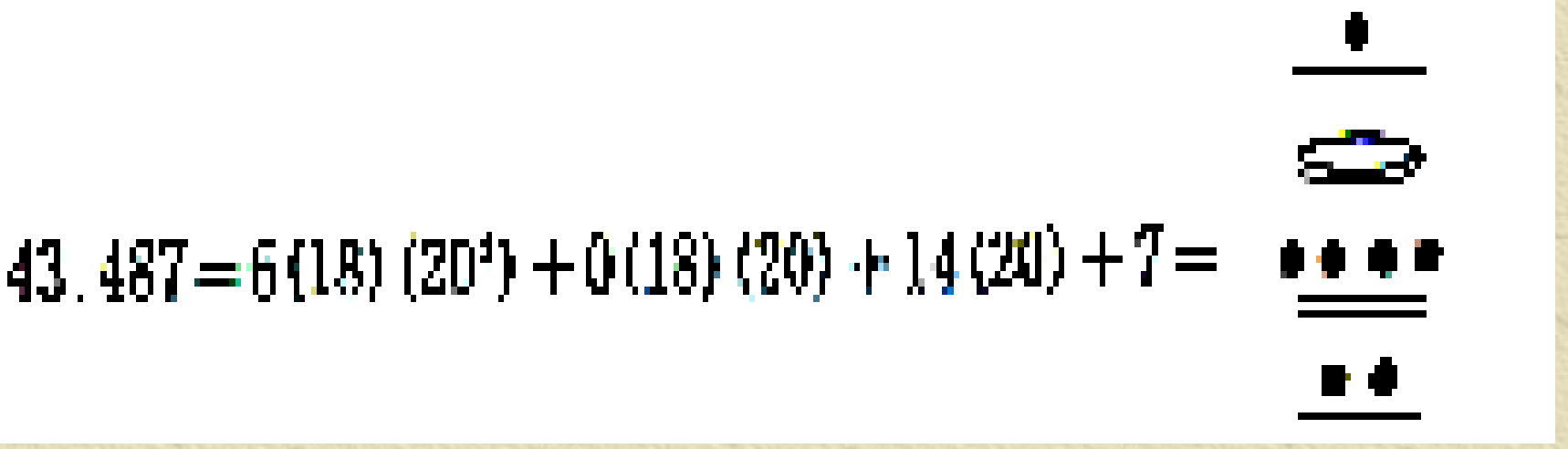
The Mayan Numeral System

The Mayan Numeral System
had a *symbol for 'Zero'* and
was written *using dots and dashes only.*



Mayan Numeral for

forty three thousand four hundred and eighty-seven



43,487 = 6 (18) (20²) + 0 (18) (20) + 14 (20) + 7

Digits are arranged from top to bottom in the decreasing order of the place-values

The Hindu-Arabic Numeral System: 1,2,3,4,5,6,7,8,9,0

The Hindu-Arabic numeral system is named after the Hindus, who have invented it, and the Arabs, who transmitted it to western Europe.

The Persian mathematician al-Khwarizmi describes Hindu System of Numerals that uses 0 (zero) and position value for digits in a book of A.D. 825.

The Hindu-Arabic Numerals
1, 2, 3, 4, 5, 6, 7, 8, 9, 0

The *Hindu-Arabic numeral system* is named after the *Hindus who have invented it*, and the *Arabs, who transmitted it to Western Europe.*

The book *al-jebr-al-muqabla* (A.D. 825) of the Persian mathematician *Mohammed ibn Musa al-Khwarizmi* describing 0 (*zero*) and *position value for digits popularized* *Hindu System of Numerals in Western urope.*

[Refer : "From One to Zero, A Universal History of Numbers" –Georges Ifrah, Viking Penguin Inc., (1985), Part VI, 26, P. 371 to 445]

Indian Zero: '0'

Indian Zero is a Multifaceted Concept

- **A Symbol : 0**
- **A Number :** ***Number of horns of a horse.***
- **A Magnitude :** ***Distance traveled by an immovable object.***
- **A Direction Separator :** ***Separates positive and negative integers on a number-line.***
- **A Place-holder :** ***In 'two thousand and five' digits of place-values of 'hundred' and 'ten' are not pronounced, but its numeral is 2005. Here missing digits of place-values 'hundred' and 'ten' are held by*** **0.**

Etymology for the Different Names of *Zero*

***Sanskrit word* for *"Zero"* is शून्य meaning *"void"* or *"empty"*.**

***Arabic Word* "sifr" is the translation of the *Sanskrit word* शून्य.**

***"Zephirum"* is the Latinized form of "sifr".**

***Leonardo Fibonooci* (1170 – 1250 AD) introduced *"sifr"* as *"cifra"* In his book *"Liber Abacci"* that describes *Hindu place-value Systems, Hindu numerals* and *arithmetic operations using them*.**

Present day word *"cipher"* for *"zero"* emerged from the *Latinized word "cifra"*

In *French "cipher"* became *"chiffre"*,

in German *"ziffer"*,

in Italian *"Zefiro"*

And finally *"Zero"*

The number 12,47,472,05,76,000 in *Sisyadhivrdhida Tantra* of *Lallacarya* (748 AD)

The circumference of the sky **in** ***yojana*** **is given as**

द्वादशलक्ष सप्तचत्वारिम्शत्सहस्रचतुःशत–
द्विसप्तति कोटयः, पञ्चलक्षः, षट्सप्ततिसहस्राणि

Twelve lakhs, forty seven thousand,
four hundred and Seventy two crores,
five lakhs, seventy six thousand.

12, 47, 472, 05, 76,000

Place-value of 2 is 2×10^{12} *and* 2×10^{7}
Place-value of 7 is 7×10^{10}, 7×10^{8}, *and* 7×10^{4}.

An Explanation for Names of Place-values
Vyasabhasya of *Yogasutra* (about 650 AD)

यथा एकरेखा शतस्थाने शतम् दशस्थाने दशैवम् च
एकस्थाने यथा च एकत्वेपि
स्त्री–माता च उच्यते दुहिता स्वसा च इति ।

Just like the same lady is addressed as mother (by her children), daughter (by her parents), daughter-in-law (by her parents-in-law), the same digit that occupy hundredth place, tenth place and unit place assumes hundredth value, tenth value and unit value.

Names of Place-values in Yajurveda Samhita *and* Taiteriya Samhita (2500 BC)

एकं च दशं च शतं च सहस्रं च अयुतं च नियुतं च प्रयुतं च अर्बुदं च न्यर्बुदं च समुद्रं च मध्यं च अन्तं च परार्धं च . . .

एकं = 1 = 10^0, दशं = 10 = 10^1, शतं = 100 = 10^2,

सहस्रं = 1000 = 10^3, अयुतं = 10,000 = 10^4,

नियुतं = 1,00,000 = 10^5, प्रयुतं = 10,00,000 = 10^6

अर्बुदं = 1,00,00,000 = 10^7 , न्यर्बुदं = 10,00,00,000 = 10^8

समुद्रं = 10^9 , मध्यं = 10^{10} , अन्तं = 10^{11} ,

परार्धं = 10,00,00,00,00,000 = 10^{12}.

Hindu Numeral System : Names of Place-values in Aryabhatiya of Aryabhata – I (5th c. AD)

एकं च दशं च शतं च सहस्रमयुतानियुते तथा प्रयुतं ।
कोट्यर्बुदं च व्रृन्दम् स्थानात् स्थानं दशगुणं भवेत् ॥

एकं = 1 = 10^0, दशं = 10 = 10^1, शतं = 100 = 10^2,

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नियुतं = 100,000 = 10^5, प्रयुतं = 10,00,000 = 10^6,

कोटि= 1,00,00,000 = 10^7, अर्बुदं= 10,00,00,000 = 10^8,

व्रृन्दम् = 1,00,00,00,000 = 10^9,

Place-value from place to place is in multiples of ten.

Names of Place-values in Ganitasarasangraha of Mahaveeracharya (815 – 877AD)

एकं तु प्रथमस्थानं द्वितीयं दशसंज्ञिकम् ।
त्रुतीयं शतमित्याहुः चतुर्थं तु सहस्रकम् ॥
पञ्चमं दशसाहस्रं षष्ठं स्याल्लक्षमेव च ।
सप्तमं दषलक्षं तु अष्टमं कोटिरुच्यते ॥
नवमं दशकाट्यस्तु दशमं शतकोठयः ।
अर्बुदं रुद्रसंयुक्तं न्यर्बुदं द्वादशं भवेत् ॥
खर्वं त्रयोदशस्थानं महाखर्वं चतुर्दशम् ।
पद्मं पञ्चदशं चैव महापद्मं तु षोडषम् ॥
क्षोणी सप्तदशं चैव महाक्षोणी दशाष्टकम् ।
शङ्खं नवदशं स्थानं महाशङ्खं तु विंशकम्॥
क्षित्यैकविंशतिस्थानं महाक्षित्या द्विविंशकम् ।
त्रिविंशकमथ क्षोभं महाक्षोभं चतुर्दशम् ॥

Sri Mahavairacarya's Ganitasarasangraha: Dr.(Mrs).Padmavatamma, Mysora, Editor and Translator, Publisher: Sri Siddhantakirti Granthamala, Sri Hombuja Jain Math, Hombuja – 577436 (karnataka) pp 18-19

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लक्षं = 1,00,000 = 10^5, दष–लक्षं = 1,00,0000 = 10^6,

कोटि = 10^7, दश–कोटि = 10^8, शत–कोटि = 10^9, अर्बुदं = 10^{10},

न्यर्बुदं = 10^{11}, खर्वं = 10^{12}, महा–खर्वं = 10^{13}, पद्मं = 10^{14},

महा–पद्मं = 10^{15}, क्षोणी = 10^{16}, महा–क्षोणी = 10^{17}, शङ्खं = 10^{18},

महा–शङ्खं = 10^{19}, क्षिति = 10^{20}, महा–क्षिति = 10^{21},

क्षोभं = 10,00,00,00,00,00,00,00,00,00,000 = 10^{22},

महा–क्षोभं = 100,00,00,00,00,00,00,00,00,00,000 = 10^{23}

Sri Mahavairacarya's Ganitasarasangraha: Dr.(Mrs).Padmavatamma, Mysora, Editor and Translator, Publisher: Sri Siddhantakirti Granthamala, Sri Hombuja Jain Math, Hombuja – 577436 (karnataka)pp.18-19

Names of large numbers in Ancient Indian Mathematics.

In the *Buddhist literature*

a centesimal scale, शतोत्तर गणन and the name तल्लक्षण for the number 10^{53} are found.

The *Jaina* work, अनुयोगद्वारसूत्र (c. 100 B.C.) mentioned large numbers up to *29 places and beyond*. The *Jainas* are further credited with the conception of a time-scale called शीर्षप्रहेलिका (84, 00, 000) and the suggestion of building up fantastically large numbers in *ascending powers of this figure*.

Reference: -*A Concise History of Science in India*: DM Bose, SN Sen, BV Subbarayappa, Indian National Science Academy, New Delhi, (1989) page 141.

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Place-value of 7 is $7x10^{10}$, $7x10^{8}$, *and* $7x10^{4}$.

Same digit has *different Names*
when it is in *different Place-values.*

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दशस्थाने दशैवं च एकस्थाने यथा च एकत्वेपि
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Vyasabhasya of *Yogasutra* (650 A.D.)

The Dispute between the Abacist and the Algorist

Finally, the abacus took backseat in 18th century.

Pierre Simon Laplace (1749 – 1827), About Hindu Numeral System

"It is India that gave us
the ingenious method of expressing all numbers by means of
ten symbols, each receiving a value position, as well as
an absolute value;
a profound and important idea
which appears so simple to us now
that we ignore its true merit,
but its very simplicity, the great ease which it has lent
to all computations, put our
arithmetic in the first rank of useful inventions,
and we shall appreciate the grandeur of this achievement
when we remember that,
it escaped the genius of Archemedes and Appollonius"

Dantzig, a Famous Historian of Mathematics About Hindu Numeral System

The long period of five thousand years *saw the rise and fall of many civilizations, each leaving behind it a heritage of literature, art, philosophy and religion.*

But what was the achievement in the field of reckoning, *the earliest art practiced by man?*

An inflexible numeration *so crude as to make progress well-nigh impossible, and a calculating devise so limited in scope that even elementary calculations called for the services of an expert.*

Man used these devices for thousands of years, without making a single worthwhile improvement in this instrument, without contributing a single important idea to the system.

When viewed In this light, the achievement of an unknown Hindu, who discovered the principle of position assumed the proportion of world achievement

Thank you