National Institute of Vedic Sciences
Certificate Course on "*Indian Mathematics*"
Syllabus for Paper II

3. Mathematics

Unit	Number of lessons	Lessons
3. 1	3. 1. 1 3. 1. 2	Numbers in Sanskrit Works; An Overview. Indian Decimal place-value system.
3. 2	 3. 2. 1 3. 2. 2 3. 2. 3	Numerals in Sanskrit Works; a) Words as Numerals b) Alphabets as Numerals and c) Early Magic Squares.
3. 3	3. 3. 1 3. 3. 2	*a) Aryabhatiya Numerals* b) Number of Revolutions of Geo-centric Planets in a Yuga (43,20,000 years) in *Aryabhatiya Numerals,* comparison of sidereal periods of Geo-centric planets in *Aryabhatiya* with their present-day values and Reason for naming the week-days.
3. 4	**3. 4. 1**	**Glimpses of Mathematics of *Sulvakaras.***
3. 5	 3. 5. 1 3. 5. 2	Glimpses of Mathematics of *Aryabhata*-I 1) *Arithmetic* and *Mensuration* in *Aryabhatiya*. *2) Circles* in *Aryabhatiya.*
3. 6	3. 6. 1 3. 6. 2	Glimpses of Mathematics of *Mahaveeracarya.*
3. 7	3. 7. 1 3. 7. 2	Glimpses of Mathematics of *Bhascara*-II.
3. 8	3. 8. 1 3. 8. 2	*Chandassutra; Zero* and Binary number System. Transmission of *Zero*, Decimal place-value system and *Indian Trigonometry* outside India Ratio of C*ircumference of a Circle to its Diameter* (π), in *Indian Mathematics*
3. 9	3. 9. 1 3. 9. 2	Biography of *Bharati Krishna Tirthaji* and Glimpses of His Contribution to *Indian Mathematics*
3. 10	3. 10. 1 3. 10. 2	Biography of *Srinivasa Ramanujan* and Glimpses of His Contribution to *Mathematics*
	20	**Total number of lessons**

Unit 3. 4 : Mathematics.

Lesson 3. 4. 1 : Glimpses of Mathematics of *Sulvakaras*

Structure: - The Unit 3. 4. 1 : Glimpses of Mathematics of *Sulvakaras* contains four parts with an introduction.

Introduction gives a brief note on the history of the development of mathematics in India and about *Sulba-sutra.*

3. 4. 1 (a) *Sulba-sutra Theorem* (Pythagorean Theorem) gives a small note on origin of the Pythagorean Theorem in several civilizations, a sloka from *Baudhayana Sulba-sutra* (600-500 BC) giving a few triplets satisfying *Sulba-sutra Theorem,* theorem of SQUARE ON THE DIAGONAL OF A RECTANGLE from *Baudhayana Sulvasutra,* and a sloka from *Katyayana Sulba-sutra* (400-300 B.C.) stating solutions of rational triangles.

3. 4. 1 (b) gives a geometrical method for determining the east-west line from *Katyayana Sulba-sutra*.

3. 4. 1 (c) gives a geometrical method to draw the perpendicular bisector of a given line from *Katyayana Sulba-sutra*.

3. 4. 1 (d) gives a geometrical method to construct an Isosceles triangle equal in area to a given square from *Katyayana Sulba-sutra*.

Objective: - The Unit: Glimpses of Mathematics of *Sulvakaras* is designed to facilitate the learner

i) To appreciate the history of the development of mathematics in India.

ii) To appreciate the Indian origin and statement of the so called 'Pythagoras theorem' in relation to that in the other civilizations.

iii) To appreciate the Indian method of drawing east-west line, its perpendicular bisector and geometrical method to construct an Isosceles triangle equal in area to a given square from *Katyayana Sulba-sutra*.

Introduction: - "The history of the development of mathematics in India is as old as the civilization of its people itself. It begins with the rudiments of metrology and computations in prehistoric times, of which some fragmentary evidence has survived to this day. The sacred literature of the Vedic Hindus - The *Samhitas*, the *Kalpasutras* and the *Vedangas* - contain enough materials, *albeit* scattered, to help form a good idea of the mathematical ability during the time of development of this class of literature. The *Sulba-sutras* which form a part of the *Kalpasutras* are a veritable storehouse of information concerning enumeration, arithmetical operations, fractions, properties of rectilinear figures, the so-called Pythagoras Theorem, surds, irrational numbers, quadratic and indeterminate equations and related matters.

The word *sulba* (*Sulva*) means a 'cord', a 'rope' or a 'string', and its root *sulb* signifies 'measuring' or 'act of measurement'. Therefore, *sulba-sutras* are a collection of rules concerning measurements with the help of a cord of various linear, spatial or three-dimensional figures. *Sulba-sutras* deal with rules for the measurements and constructions of various sacrificial altars and consequently involve geometrical propositions and problems relating to rectilinear figures, their combinations and transformations, squaring the circle, circling the square as well as arithmetical and algebraic solutions of problems arising out of such measurements and constructions. The *Baudhayana Sulba-sutra* is the oldest (600-500 B.C.). The *Manava Sulba-sutra* is posterior to *Baudhayana* and contains descriptions of a number of altars, not found in other works. The *Apastamba Sulba-sutra* (500-400 B.C.) gives the same rules as were found in *Baudhayana Sulba-sutra.* The *Katyayana Sulba-sutra* (400-300 B.C.) is more succinct and more systematic.

3. 4. 1 (a) Sulba-sutra Theorem (Pythagorean Theorem)

"The question of the Greek origin of the Pythagorean Theorem and also whether Pythagoras himself was the discoverer of it and its proof has by no means been solved. The tradition attributing the theorem to Pythagoras is due to Cicero (c. 50 B.C.), Diogenes Laertius (2nd c. A.D.). Athenaeus (c. A.D. 300), Heron (3rd c. A.D.) and Proclus (c. A.D. 460), and therefore started about five centuries after the death of Pythagoras. Junge pointed out that the Greek literature of the first five centuries after Pythagoras contained no mention of the discovery of this or any other important geometrical theorem by the great philosopher and further more emphasized uncertainties in the statements of Plutarch and Proclus. Although various attempts have been made to justify the tradition and trace the proof to Pythagoras, no record of proof has come down to us earlier than that given by Euclid (Theorem 47, BK I).

As to the relation $3^2 + 4^2 = 5^2$ from which the theorem of rational triangle is derivable, very ancient Egyptian knowledge is attested by the Kahun papyrus of the twelfth dynasty (c. 2000 B.C.), but its association with rational triangles does not seem indicated in this or other Egyptian papyrii. It is interesting to note that among the Egyptians, geometry of surveying was considered to be the science of the 'rope-strechers' who thus appear to be the Egyptian counterpart of the Indian *Sulbavids.* As to the antiquity of Pythagorean theorem in China, it is stated, though not proved, in the arithmetical classic *Chou Pei Suan Ching* (3rd or 4th c. B.C.); the numerical relationship 4, 3 and 5 between the sides and the diagonal of a rational rectangle is also given in this text."

[Ref: *A Concise History of Science in India*, D. M. Bose, S. N. Sen, B. V. Subbarayappa [Editors] INSA, New Delhi p. 149]

A question has often been asked whether such a definition resulted from empirical guesswork or was based on a proof of some kind. . . . There is hardly any doubt that the vedic *sulbavids* possessed a valid proof of the theorem, of which the texts themselve provide reliable indications". For example a rule in *Baudhayana Sulba-sutra* states;

तासाम् त्रिकचतुस्कयोर्द्वादशिकपञ्चिकयोः पञ्चदशिकाष्टिकयोः सप्तिकचतुर्विंशिकयोर्
द्वादशिकपञ्चत्रिंशिकयोः पञ्चदशिकषत्त्रिंशिकयोरित्येतासूपलब्धिः ।

"This is observed in rectangles having sides 3 and 4, 12 and 5, 15 and 8, 7 and 24, 12 and 35, 15 and 36".

The sum of squares of the numbers stated in the previous sloka is also a square number for each pair thus;

$$\mathbf{3^2 + 4^2 = 5^2,\ \ 12^2 + 5^2 = 13^2,\ \ 15^2 + 8^2 = 17^2,}$$
$$\mathbf{7^2 + 24^2 = 25^2,\ \ 12^2 + 35^2 = 37^2,\ \ 15^2 + 36^2 = 39^2.}$$

A general expression for obtaining pythagorean triplet is

$$\boldsymbol{(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2}$$

such that $x^2 + y^2 = z^2$, where $x = (m^2 - n^2)$, $y = (2mn)$, and $z = (m^2 + n^2)$.

When $m = 2$ and $n = 1$, $\mathbf{3^2 + 4^2 = 5^2}$; When $m = 3$ and $n = 2$, $\mathbf{5^2 + 12^2 = 13^2}$;

When $m = 4$ and $n = 1$, $\mathbf{15^2 + 8^2 = 17^2}$; When $m = 4$ and $n = 3$, $\mathbf{7^2 + 24^2 = 25^2}$.

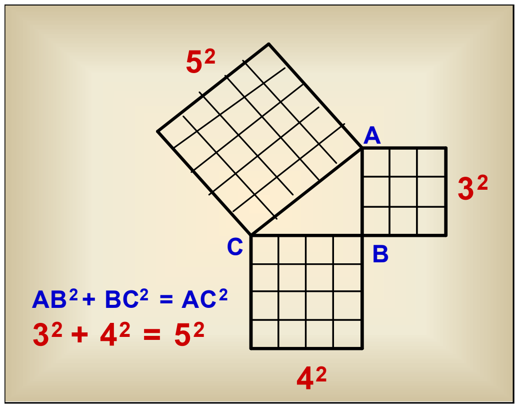
When $m = 6$ and $n = 1$, $\mathbf{35^2 + 12^2 = 37^2}$.

But $\mathbf{15^2 + 36^2 = 39^2}$ is derived from $\mathbf{5^2 + 12^2 = 13^2}$, multiplying it by $\mathbf{3^2}$.

The above relationships between sides and diagonals or hypotenuse for rational rectangles have been freely used for finding the perpendicular directions (north-south) at the east or the west point of the east-west line in altar settings.

The method is to take a cord of suitable length greater than the distance between the two poles one each at the east and the west point, fasten a loop at either end of the cord, fix it with the two poles by these loops and then stretch it by the *niranchana* mark that divides the cord in two portions x and y so that $x^2 + y^2 = z^2$.

This finding of the properties of rational triangles is a strong argument in favour of the Vedic Hindus possessing proofs of the theorem of the square of diagonal or the hypotenuse. It is possible that by actually drawing the squares on the diagonal and the sides of a rational triangle, dividing them into elementary unit squares and then counting them, they might have arrived at the truth of the theorem.

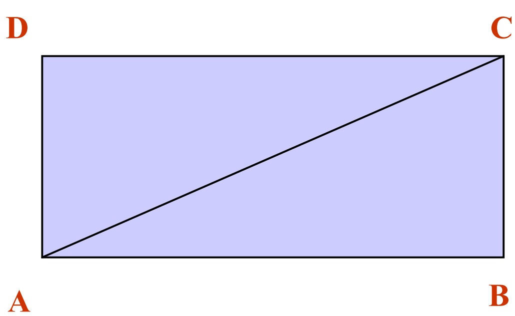


Theorem of SQUARE ON THE DIAGONAL OF A RECTANGLE –

Baudhayana Sulvasutra (600-500 BC)

दीर्घचतुरस्रस्याक्ष्णया रज्जुः पार्श्वमानी तिर्यङ्मानी च
यत्पृथग्भूते कुरुतस्तदुभयं करोति ॥१. १२॥

The Areas (of the squares) produced separately by the length and the breadth of a rectangle together equal the area of the square produced by the diagonal.



$AB^2 + BC^2 = AC^2$

The *Katyayana Sulba-sutra* (400-300 B.C.) gives solutions of rational triangles and states the pythagorean Theorem in a generalized form thus;

दीर्घचतुरश्रस्याक्ष्णया रज्जुस्तिर्यमानी पार्श्वमानी च यत्पृथग्भूते कुरुतस्तदुभयम् करौति क्षेत्रज्ञानम् ।

"The (area of the) square drawn on the diagonal of a rectangle is equal to the sum of (areas of) the squares drawn seperately on its breadth and length; this is the property of plane figures (concerning rectangles)".

The word इति क्षेत्रज्ञानम् at the end, meaning 'this is the knowledge of plane figures' suggests solutions of rational triangles and states the Pythagorean Theorem in a generalized form. '*To speculate on whether the Indians had a proof for the theorem or what the proof could have been is idle. The sulbasutras . . . are only practical manuals for the construction of the altars. Proofs are outside their scope. Very likely they had proofs orally transmitted to the enquiring student.*

The constructions dealt with in the Sulbasutras comprise the construction of east-west line; of perpendiculars; of squares, rectangles, and trapezia ; and of triangles and rhombi equal in area to a given square; conversion of squares into rectangles and vice versa; of squares into circles and vice versa.' [Refer : *Geometry in Ancient and Medieval India* : Dr. T.A. Sarasvati Amma, Motilal Banarsidass Publishers (1999), p.21-45]

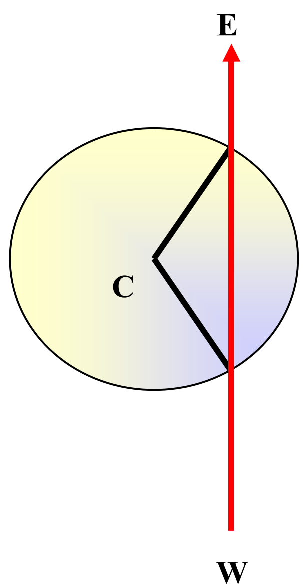
3. 4. 1 (b) Determining the east-west line

This was preliminary to the construction of all the altars and sacred pyres described in the Vedic literature. But it is only *Katyayana* and *Manu* that give the details of the procedure.

Katyayana states the procedure thus ;

समे शङ्कुं निखाय शङ्कुसम्मितया रज्वा मण्डलं परिलिख्य यत्र लेखयोः शङ्क्वग्रच्छाया निपतति तत्र शङ्कु निहन्ति सा प्राची ॥१.२॥

Having put a pole on a level ground and described a circle round it by means of a cord (*fastened to the pole*), *a pole is fixed on each of the two points where the end of the pole's shadow touches* (*the two halves of the circle*). *This line joining the end of the pole's shadow is the east-west line* (*praci*).



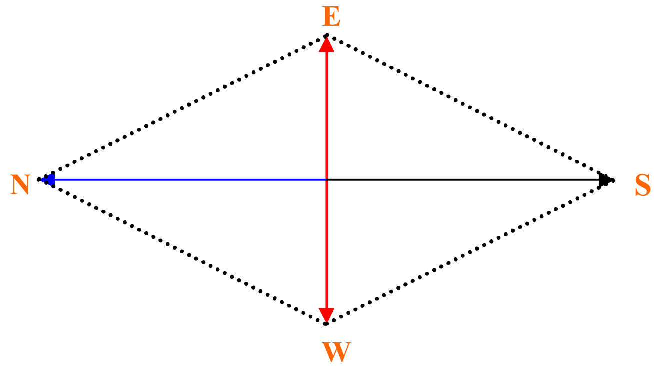
3. 4. 1 (c) To draw the perpendicular bisector of a given line

Katyayana states the procedure thus ;

समे शङ्कुं निखाय शङ्कुसम्मितया रज्वा मण्डलं परिलिख्य

यत्र लेखयोः शङ्क्वग्रच्छाया निपतति तत्र शङ्कु निहन्ति सा प्राची ॥१.२॥

Then after doubling (a given) *cord, two loops* (made at its two ends) *are fixed at the two poles* (of the *Praci*), *and the cord is streched towards south by its middle point where a pole is fixed ; the same is repeated to the north. This line joining the two poles is the north-south line* (*Udici*).



This is the same as the modern method of drawing the perpendicular bisector of a line. Only, instead of drawing intersecting arcs to get two points equidistant from the ends of the line, isosceles triangles are drawn on either side of the line with the as the base and their vertices are joined.

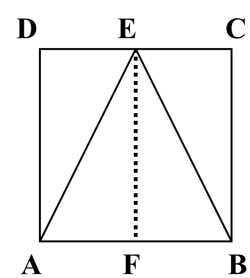
3. 4. 1 (d) To Construct an Isosceles Triangle equal in Area to a given Square:

Conversion of a square into an Isosceles Triangle equal in Area to a given Square, being necessary for the construction of the *praugacit,* is tackled by all the three important *Sulvasutras* and all of them give the same prescription.

Apastamba-sulvasutra states ;

यावानग्निः सारत्निप्रदेशो द्विस्टावतीम् भूमिं चतुरश्रम् कृत्वा पूर्वस्याः करण्या अर्धात् श्रोणीप्रत्यालिखेत् ।
सा नित्या प्रौगम् ।

A square twice as large as the area of the fire-altar with aratnis *and* pradesa *is laid; the mid-point of the eastern side of the square is joined to the two western corners of the square, and the area lying outside these lines is cut-off ; this is the exact Isosceles triangle.*



Let ABCD be a square of twice the required area. Let E be the middle point of CD. EA and EB are joined. Then AEB is the required triangle. For, if the altitude EF is drawn, the square is divided into 2 equal rectangles AFED and FBCE.
Area of *triangle* AFE = Area of *half of rectangle* AFED, and
Area of *triangle* FBE = Area of *half of rectangle* FBCE.
Therefore, Area of *triangle* AEB = Area of *half of Square* ABCD
This construction leads to the formula;
Area of *triangle* = *half of* (base x altitude).

Exercise: -

1. Write a short-note on *Sulba-sutra.*
2. What is the difference between the *present day method* and *Sulba-sutra method* of drawing a vertical line and drawing the perpendicular bisector of a line segment?

3. What are the different reasons for Greeks and Indians to develop Geometrical concepts?
4. What evidence confirms the knowledge of ancient Indians about *'the square on the hypotenuse of any rational right triangle'?*