A Skit on **Bhaskaracarya** – II (12th c. A.D.)

900th birth anniversary of BHASKARA II

THE PEARL

A Skit on Bhaskaracarya-II, a renowned Mathematician (born 1114 AD)

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Introduction:-

The pearl, a skit on Bhaskaracharya – II is devised and enacted on several occasions, to bring to light a glimpse of Acharya's ever popular works. <u>The skit is based on a popular folk tale about the Acharya</u>. *His mathematical poems* incorporated in the skit are *to the level of understanding of secondary and senior secondary students*. These lyrical poems are set to classical tunes. <u>Each scene of the Skit commences with narration giving information about the works of Bhaskaracarya and related matters</u>.

- i) The skit was staged Sainik School, Amaravatinagar, Tamilnadu and Kendriya vidyalayas of Bhubaneswar and Jalahalli several times as a part of cultural programmes on several prime functions.
- At Gayana Samaja, K.R.Road, Bangalore on 20th Sept 2004. Kannada version "Muthinamani" of "The Pearl" as a part of cultural programme during a three day Symposium on Vedic Sciences conduced by National Institute of Vedic Sciences, Mulabagal from 20th 22nd September 2004. It was enacted by students of International Academy for Creative Teaching, Hebbal, Bangalore in which I was Dean (Mathematics).
- iii) The skit was staged by the student-teachers and staff of International Academy for Creative Teaching, during a weekly programme of 'Bangalore Science Forum' (October 2004) in the presence of Prof. Dr. H. Narasimhaiaha, a famous educationist, a true Gandhian and an ex vice-Chancellor of Bangalore University, and got appreciation. Prof. Dr. H. Narasimhaiaha conducted weekly programme of 'Bangalore Science Forum' every Wednesday, since 1964 in the famous National College, Bangalore where he lived in a small hostel room throughout his life, even when he became Vice-Chancellor and Chairman of Kannada Development Authority and received a token remuneration of Re. one per month.

iv) A 'Two episode' lecture on the same topic supported by the Professional singers and Instrumentalists along with the projection of slides projected through OHP was recorded and *Telecast, twice* by *Bangalore Door Darshan*, under the programme '*Samskrita aurabha*' during 1999 and 2000, and being telecast periodically.

THE PEARL

Narration 1:- (Two narrators, a lady and a gentleman enter in. Both of them greet the audience.)

Both :- Namaskar, Ladies and Gentleman, we wish you all a pleasant evening and welcome you all with reverence . Let us witness a choreography of the invocation and the autobiographical poem of Bhaskaracharya-II

Udayaraaga

उत्पादकं यत्प्रवदन्ति बुद्धेर्अधिष्ठितं सत्पुरुषेण सांख्याः । व्यक्तस्य कृत्स्नस्य तदेकबीजम्अव्यक्तमीशं गणितं च वन्दे ॥१॥

- [Ref.1 (p.2)]

- Narrator 1:- Will you please tell me the meaning of invocation sloka ? Oh! It has words like "Sankhyaa, ... GaNitha". I couldn't make out anything from the sloka.
- Narrator 2:- Alright Its meaning is "I bow with reverence to that Unmanifested, which the wise regard as the sub-stratum of the being the source of intelligence and the root cause of this world and to the Mathematics , which has similar attributes." This is the first sloka in '*BijagaNita*' of Bhaskaracarya, a part of 'Sidhantha Shiromani' written in 1150 AD.
- Narrator 1:- I've heard that Indian History isn't accurate. Indians weren't keeping accurate records of events happened.
- Narrator 2:- But, Bhaskaracarya has stated about his illustrious father, place of birth, year of his birth and the year durng which he penned his treatise '*Sidhantha Shiromani*'. Let's hear the sloka.

2. Autobiographical sloka - Raag : Bhairavi आसीत्सहयकुलाचलाश्रित–पुरे त्रैविद्यविद्वज्जने । नानासज्जनधाम्नि बिज्जलबिडे शाण्डिल्यगोत्रो दि्वजः ॥ श्रौतस्मार्तविचारसारचतुरो निःशेषविद्यानिधिः । साधूनमवधिर्महेश्वरकृती दैवञचूडामणिः ॥ तत्सज्जनचरणारविंदयुगलप्राप्तप्रसादः सुधीः । मुग्धोद्भोधकरं विदग्धगणकप्रीतिप्रदं प्रस्फुटम् ॥ एतद् व्यक्तसदुक्तियुक्तिबहुलं हेलावगम्यं विदाम् । सिद्धान्तग्रथनं कुबुद्धिमथनं चक्रे कविर्भास्करः ॥ रसगुणपूर्णमहीसमशकनृपसमयेऽभवन्ममोत्पत्तिः । रसगुणवर्षेण मया सिद्धांतशिरोमणी रचितः ॥

- Narrator 1:- Beautiful ! ... Marvelous! A lyrical mathematical sloka. I shall tell you its meaning. In a place named Bijjalabida, located near the Sahyadri range of hills, lived a renowned scholar named Maheswara. He was born to a noble clan of Shandilya and he was proficient in Vedic rituals. He was praised for his gentleness, piety and blissful intelligence. Having born to such an illustrious father, I acquired the nectar of mathematical knowledge through him as a precious boon. I used this precious boon of knowledge to pen lyrical mathematical poems, to confound the rivals, and to educate the knowledge. seekers of And thus achieved the laurels as KAVIRBHASKARA." ... This is its meaning. But, ... there is no mention of any number referring to his year of birth or anything else.
- Narrator 2:- The sentence

रसगुणपूर्णमहीसमशकनृपसमयेऽभवन्ममोत्पत्तिः । [Ref. (3) (P. xvi] says that 'I was born in the year 1036 of Vikrama shaka', because रसगुणपूर्णमही stands for **1036.** रस - stands for **6**, गुण – thriguna stands for **3**, पूर्णम् for *Zero* and *infinity*, but when it is used as a digit, पूर्णम् denotes *Zero*, महि– one, the only inhabitable Earth. As per the rule अङ्कानां वामतोगतिः the (place-values of) digits (in the numeral of a number) moves (increases) towards left.

Narrator 1:- Now I remember, the calendar based on विक्रमशक, which is also the national calendar commenced form 78 AD commemorating the year of coronation of king Shalivahana. Accordingly Bhaskaracharya - II was born in the year 1114 AD and

रसग्णवर्षेण मया सिद्धांतशिरोमणी रचितः ॥ [Ref. (3) (P. xvi]

means; "I penned Sidhantha Siromani when I was 36 year old" (in 1150 AD).

Narrator 2:- You're absolutely correct. As a boy, Bhaskara brooded over the ills of the society around him. The adamant wall of customs had always been a source of pain to him. But earlier works comforted young Bhaskara by revealing that people had once lived free and happy life and he found solace in imagining such a free society in days to come.

Scene 1

(Early morning. Twelve-year-old Bhaskara is plucking flowers for pooja in the garden. White lotus flowers in the pond nearby attract his attention. He goes near the pond and stands for-a-while)

Bhaskara: (thinks aloud) Oh, . . . spotless white lotus, . . . how beautiful, . . . really enchanting, . . . spotless white lotus,

(In the meanwhile, his friend Sayana, calling "Bhaskara, ... Bhaskara" enters. Observing Bhaskara absorbed in gazing the pond stands for a while, then goes near him and gently touches him.)

Sayana: Bhaskara, ... Bhaskara (*Bhaskara looks at him*) Aren't we late for pooja? Come on let us go (*Holds his hand and tries to lead him*.)

Bhaskara: (*Shirking his hand*) Wait, Sayana ... listen,.. These spotless white lotus inspired me to compose a poem... wait a minute. (*Stands still near the pond, after a while*) Tell me whether Raag Desh suits this poem.

Sayana: Are you going to sing the poem you have just composed? (*With pleasure*) Excellent, come on my friend.

(By then, Maheswaropadhyaya and his two disciples Ahobala and Langoola had entered from behind. After hearing Sayana pleading Bhaskara to sing, Maheswara and the two disciples hide behind a tree and listen.)

Bhaskara:

Raag - Desh

 अमलकमलराशेस्त्र्यंशपञ्चांशषष्ठैः त्रिनयनहरिसूर्या येन तुर्येण चार्या गुरुपदमथ षड्भिः पूजितं शेषपद्मैः सकलकमलसङ्ख्यां क्षिप्रमाख्याहि तस्य ॥LVII [Ref. {(3) (P.57)}]

- Sayana:Marvelous, The poem is on a mathematical problem Beautiful..... Why
don't you show all your poems to Revered Acharya? How glad he would be?
- Bhaskara: (*Morose*) I don't think so..... Don't you know how he snubs me, whenever I say anything, which I feel right?.... Has he ever encouraged my freethinking?
- Sanya: (consoles) You are mistaken, Bhaskara... Revered teacher loves you so much. He fears the people, who may harm you by mistaking your free thoughts. Also aren't you aware that he is strictly against anyone criticising the established norms and customs?
- Bhaskara: (*diverts the conversation*) Any way you may be right, leave it... Have you followed the problem in the poem?

Sayana: Why not? The problem is to find the number of white lotus collected for pooja from which one-third of its number offered to Lord Shiva, one-half to Maha

A Skit on **Bhaskaracarya – II** (12th c. A.D.)

Vishnu, one-sixth to the Sun God, one-fourth to Goddess Parvathi and the remaining six at the lotus feet of revered Acharya.

(Ahobala and Langoola approach Bhaskara. Maheswara moves towards plant bearing flowers and starts plucking them.)

- Ahobala: Bhaskara, there must be one hundred and twenty flowers... Your poem is really marvelous. It excels all mathematical poems written so far.
- Langoola: (*ridiculing*) marvelous, ... beautiful, ... It might please only you. I don't find anything special in it. Why should anyone compose MATHEMATICAL POEMS?
- Ahobala: (*approves*) You are right Langoola... Haven't we heard, 'Prose form is the appropriate one to pose problems in mathematics.'
- Maheswara : -(*Maheswara moving towards them*) Ahobala, what makes you say that there were 120 flowers taken for the puja?
- Ahobala: Guruji, out of one heap of flowers collected for puja, one-third offered to shiva and one-fifth to Vishnu adds to eight-fifteenth. One-sixth offered to Sun god when added to the sum gives seven by ten. When this is added to the one-fourth offered to Goddess Parvati the total becomes nineteen by twenty. Remaining onetwentieth of heap is offered to the Guru and that is equal to six flowers. Therefore, there were 120 flowers in the heap collected for puja.
- Langoola: Any way, let us hope this new indulgence of Bhaskara saves our sacred scriptures from vilifying comments and criticisms by him.
- Ahobala: Are you referring to his yesterday's criticism on Revered Acharya's commentary on....(*Maheshwara looks at them quizzically.*)
- Langoola: It is true Acharya. Bhaskara decried the meaning of the sacred sloka

ब्राहमणोस्य मुखमासित् बाहूराजन्यकृतः।

'Brahmins are born out of the face of the Lord; Kshatrias, the warriors out of His shoulders; Vaishyas, the Merchants out of His thighs and Shoodras, the labourers out of His feet:

Bhaskara: (When Acharya looks at him with surprise, he explains after paying respect with folded hands) I never decried the meaning of the sacred sloka Acharya. I said that, apparent meaning of the sloka is only a simile.... But its real meaning is 'all those who utilise usefully, the vital sensory organs in the face, to educate themselves and also all the people in the society are Brahmins; and those who use the strength of their shoulders to defend and expand their territories are warriors the Kshatrias....

The Pearl A Skit on **Bhaskaracarya – II** (12^{th} c. A.D.)

- Langoola: *(interrupts and imitates)* All those who toil in the fields to grow food are Shudras and they are the supporters of the entire society and they should also be educated.... These are the revolutionary ideas of our Bhaskara... Is he right in vilifying your sacred commentary, Acharya?
- Maheswara: (*introspection*) Right, . . .wrong, . . . Customs and norms are based on values. . . .
 Values change as the time passes. . . . A belief accepted as right today, might be proved wrong in future, based on changed values.Bhaskara, you must learn to live in the present. You are talented in rendering mathematical poems in lyrical slokas. Cultivate this talent. Try to render mathematical theories also in lyrical slokas. ... Let us move.... We are late for pooja. (*seeing Bhaskara trying to say something*) Bhaskara, ask me. ... Don't hesitate .

Bhaskara: Sir, what is the result from the division by zero?

Maheshvara: Revered Sages have forbidden the division by Zero? Don't ask such questions. ... Think yourself. ... you will realise.

Curtain Down

Narration 2 :- (*the two narrators enter*)

Narrator 1: What a pity ! ... Acharya should have encouraged Bhaskara

Narrator 2: Do you know ? Similar thing had happened to Ramanujan.

- Narrator 1: Are you referring to Ramanujacharya, the sage ... propounder of Vishishtadvaita Philosophy.
- Narrator 2: No. ... I referred to the mathematician Srinivasa Ramanujan. When he was a student in a primary class, his teacher posed a question about the division of 12 mangoes among 4 students. Ramanujan asked the teacher about the division of 12 mangoes among no student. Isn't this question similar to that by Bhaskara ?
- Narrator 1: Certainly. ... Ramanujan must have had a good thrashing. (Laughs)
- Narrator 2: I'm not sure. ... But the teacher was certainly embarrassed. ... Let us move.

(Exit)

Scene – 2

(In a prominent place of Mahesvaracharya's ashram, under a peepul tree, lies an elevated platform on which a deer-skin (Krishnajina) is spread, over which a Vyasapeetha is kept.Smooth grass mats are spread in front of the platform for students to be seated. A pond filled with spotless white lotus flowers is near-by. Bhaskara seated alone on a seat nearer to the pondis busy in writing on a palm leaf. Sayana enters, sits by the side of Bhaskara, waits untilBhaskar finishes writing. Bhaskara finishes writing and looks at Sayana.)

A Skit on *Bhaskaracarya – II* (12th c. A.D.)

Sayana: What are you writing Bhaskara?

Bhaskara: (handing over the palm-leaf on which he finished writing) Sayana, read this.

Sayana: (reads aloud) खहरस्यात् खभाजकोराशिः ।

(in the meanwhile other students were already seated and had heard Sayana reading Bhakara's writing)

Langoola: (laughs aloud खहरस्यात् खभाजकोराशिः ।

(all the students except Sayana and Bhaskara laughs loudly. Acharya enters. Students stop laughing, offers pronams by reciting pravara traditionally while standing. All of them sit after Acharya being seated)

Acharya : Langoola, what made you to laugh so loudly ?

Langooola: Acharya, have you not warned us about division by Zero?

Acharya: Yes. ... But ...

Ahobala: But Bhaskara says 'a number divided by zero is a fraction whose divisor is zero.' Have'nt you told us 'the division by zero is forbidden.'

Maheswara: Langoola, can you explain why 'the division by zero is forbidden.'

- Langoola : Achaarya, quotient of an improper fraction 18 by 6 is 3, because the denominator, that is the divisor 6 is to be added to itself 3 times to get its numerator 18.Similarly, the quotient of an improper fraction 18 by 0 is the number of times the denominator 0 is to be added to itself to get its numerator 18. But, whatever may be the number of times 0 is added to itself, the sum is always Zero, but never its numerator is 18, or any other number. Therefore, *the division by zero is forbidden*.
- Maheswara: Well said Langoola, a brilliant explanation, yes, . . . division by zero is forbidden.(*Smiling*) Bhaskara, is it not childish to say 'a number divided by zero is a fraction whose divisor is zero'?
- Bhaskara: (*with respect*) I shall explain, if you permit Acharya. A fraction with denominator zero is only a numeral or a symbol of the value obtained while dividing any number by zero. But the value of such a fraction is infinite, it is only a concept but not a number, in the real sense.
- Ahobala: (*appreciating*) How could that be merely a concept, but not a number? Will you please explain?

Bhaskara: Surely, what is the quotient when 12 is divided by 6?

Ahobala: Two.

Bhaskara: When 12 is divided by one upon one-lakh?

Langoola:	(ridiculing) Twelve lakhs. What a new theory you developed Acharya	
	(observing Acharya absorbed in the conversation, keeps quiet)	
Sayana:	(exited) I followed it Bhaskara, 'when numerator is kept constant and denominator	
	is decreased, the value of quotient increases. (Doubt) What has this to do with a	
	fraction whose denominator is Zero?	
Bhaskara:	You shall see it When you compare Zero with a fraction having a very huge	
	number in its denominator, which is the smallest out of the two?	
Langoola:	Zero is the smallest. (Ridiculing) Don't you know even this simple fact, Bhaskara?	
Maheswara:	(musing) Zero is the smallest compared to a fraction having a very huge number in	
	its denominator. Well said Langooola. Could you explain it clearly?	
Langoola:	I shall, Acharya (bows with respect). If a very huge object could be divided into one	
	million parts, one such millionth part is surely something, which exists. It is not	
	Nothing or Zero. Therefore Zero is the smallest of all the fractions having very	
	huge numbers in their denominators.	
Bhaskara:	You are right Langoola, That is why Zero could be considered as a fraction having	
	a very huge number in its denominator, whose hugeness is beyond the	
	comprehension of any human being. Hence a number divided by zero must have	
	such a huge value. Its value is infinite. It cannot be a number but only a concept.	
Maheswara:	(emotion) Bhaskara, you are really a genius Have you thought of any property	
	of this infinite value?	
Bhaskara:	(respect) I have written a poem explaining its property. (Sings in Raag - Kalyan)	
	4. अस्मिन् विकारः खहरेन राश्यावपिप्रविष्टेश्वपि निःसृतेषु ।	
	बह्ष्वपिस्याऽल्लयसृष्टिकालेऽनन्तेऽच्युते भूतगणेषु यद्वत् ॥६॥ [Ref. 1(p.18)]	
Maheswara:	(appreciating) Aha, aha, what a beautiful explanation 'The value of the	
	fraction whose denominator is Zero would never change even when many are	
	added to it or subtracted from it, as in the case of inimitable God at the period of	
	destruction or creation of the world, even though numerous orders of beings are	
	absorbed or put forth by the Almighty.' What an appropriate simile	
	Bhaskara, I am proud of you my son Let your noble venture of rendering	
	difficult mathematical theories and problems into beautiful lyrical poems be	
	continued May the Lord bless you in your attempt to popularize mathematics.	
Curtain Down		

Narration 3:

- Narrator 1 : "The value of a fraction whose denominator is zero, is infinite, and it is not affected if many are added to it or subtracted from it" this statement of Bhaskara II has suggested that he is the one who sowed the seed for the idea of limit and thus to calculus.
- Narrator 2: Bhaskara Acharya acquired mastery in rendering dry mathematical concepts as rhythmic poems. He was acclaimed as 'KavirBhaskara'. He was honored as one of the best astronomers and mathematicians.
- Narrator 1: Ancient mathematicians had already made remarkable contributions to astronomy and other branches of sciences. They had developed mathematical rules to solve problems in these applied sciences.
- Narrator 2: Unlike his prodecessors, Bhaskaracharya started collecting rules to solve problems in arithmetic, mensuration and algebra and also devised pure mathematical problems from the experience of day to day life.
- Narrator 1: Bhaskaracharya was appointed as a professor in the astronomical observatory of Ujjain at a very young age.
- Narrator 2: Oh, Acharya seems to be anxious about something. He appears to be restless. Come on, let's know the reason.

Scene 3

(Morning, Acharya is seen pacing up and down with anxiety. Maheswaracharya enters.)

- Maheswara : I had sent words to Royal Astrologers. The nurse says delivery is expected any time now. But, . . .
- Bhaskara: (gets up) The Royal Astrologers might be on the way. I shall send some one to receive them. (goes out)

(Bhaskara returns accompanied by Dixitar and Pranesha Acharya. The two astrologers pay respect to the Acharya, the elder, traditionally by reciting pravara)

- Maheswara: May God bless you. Please be seated. (*After they sit*) Bhaskaracharya is expected to become father any moment from now. I desire you to set the horoscope of the newborn, soon after you hear its first cry. Excuse me for the trouble.
- Dikashitar: Is it a trouble? . . . Not at all You have blessed us by providing this rare opportunity.
- Pranesha: What else, . . . aren't we aware you are an authority on astrology We are grateful to you sir.

The Pearl A Skit on **Bhaskaracarya – II** (12^{th} c. A.D.)

Maheswara:	Your talk manifests your humility. May God bless you. Do we set horoscope of	
	our own family members?	
Dikshitar:	Never, it is against professional ethics for the people like us. But, is it applicable	
	to a noble person like you Sir?	
Pranesha:	Bhaskaracharya, when I had been to palace this morning I was instructed to	
	convey you the desire of, His highness the Maharaja, to meet you at the earliest.	
Bhaskara:	Thank you for the message Acharya Appaji, shall I	
(When he was about to take leave, cry of the newborn baby is heard. Astrologers engage themselves in		
setting the horoscope of the newborn baby. Bhaskara eagerly waits for the news)		
Pranesha:	(looks at Dikshitar and after getting a nod of approval) The newborn baby is	
	(hesitates)	

Dikshiter: The baby is a girl.

(Maheswara looks disturbed. In the meanwhile, the maid-servant enters and announces)

Maid-servant: Sir, the revered lady is blessed with a female child Both are healthy.

Bhaskara: (*with pleasure*) A girl, Goddess Mahalaxmi, . . . Goddess Laxmi is born to bless the abode of Saraswati, the Goddess of learning I am really blessed (takes out a golden ring from his finger and presents it to the maid-servant). Convey my gratitude to your revered lady. ...

(Maid servant leaves. The astrologers continue with their calculations for setting the horoscope. Maheswara and Bhaskara observe them anxiously. After-a-while)

Dikshitar: Sir, your grand-child would be a great scholar, ... But

Maheswara: But?... Tell me, why do you hesitate? (Anxiety)

Pranesha: Her married life,

Maheswara: (stunned) Her married life?...

Diksitar: would be very much short.

(Maheswara stands thunder-struck. As astrologers move towards him to console him, darkness spreads.)

Curtain Down

Narration 4:

- Narrator 1: Lilavati, the beloved daughter of Bhaskaracharya grew up as a charming girl. Progressive minded Acharya taught her mathematics, philosophy, epics, scriptures and literature.
- Narrator 2: Acharya, who considered astrology as an applied science of astronomy, pondered over the prediction of astronomers about the short span of Lilavati's married life. He decided to bring her up as a scholar.

- Narrator 1: Acharya improved upon the ancient mathematical theories His contribution in giving general methods to solve indeterminate equations are praise worthy.
- Narrator 2: Friend, I have a doubt. Irrational numbers like $\sqrt{14}$ cannot be used as such in calculations. It is necessary to know its rational approximations. How did the Indian mathematicians overcame this difficulty ?
- Narrator 1: Indian mathematicians adopted different methods through centuries. Among them *Bhavana* process of Brahmagupta invented during 6th century AD is noteworthy.
- Narrator 2: Will you please explain.
- Narrator 1: 'Bhavana' process gives approximate value of such an irrational number, whose square would become a square number only when 4 or 2 or 1 is added to it or subtracted from it. Indian mathematicians defined an equation involving square of such an irrational number \sqrt{N} *i.e.*, *N* and named it *prakruti*.
- Narrator 2: What is Vargaprakruti?
- Narrator 1: To find rational approximation to any irrational number \sqrt{N} , they formed an equation $Nx^2 + h = y^2$ and named the equation *Vargaprakruti*. They noticed

$$\sqrt{N} = \sqrt{\frac{y^2 - h}{x^2}} = \frac{y}{x}(app.)$$
 and they attempted to find integral values of x and y,

making h a very small positive integer 1. 'Bhavana' process is one of the attempts to get approximate values of an irrational number, whose square is a square, only when 4 or 2 or 1 is added to it or subtracted from it.

Narrator 2: Thank you for this thrilling information.... Acharya seems to be teaching his daughter Lilavati. Come on, let's move.

Scene 4

(Morning time. Thirty-five year old Acharya and thirteen year old Lilavati are seated in the study room.)

- Bhaskara:Lilavati, tomorrow let us discuss the final battle between Karna and Partha. . . .What is your opinion about the episodes in the epic discussed so far?
- Lilavati: It is very unfortunate Appaji, . . . Our epics always describe battles, blood-shed and enmity between brothers How fine it would have been if Karna were to convince his friend Suyodhana for the treaty with Pandavas.

Bhaskara: You are right.... But, the epics mirror real life... They also describe certain qualities which are very rare in the real life, which manifests at times Karna is the one who possessed such rare qualities... He rejected the tempting offer to rule as a Lord acceptable to both the sides, but sacrificed his life for the sake of true friendship.... Partha, the real brother of Karna, killed Karna... (becomes emotional) Parthah, ... Karna Vadha, ... Parthah Karna Vadhaaya, ... Parthah Karnavadhaaya Maarganaganam ... (thinks, then sings melodiously)

Raag Hamsanandhi

- पार्थः कर्णवधाय मार्गणगणं कृद्धो रणे संदर्ध । तस्यार्धन निवार्य तच्छरगणं मूलैश्चतुर्भिर्हयान् । शल्यं षड्भिरथेषुभिस्त्रिभिरपि च्छत्रं ध्वजं कार्मुकं । चिच्छेदास्य शिरः शरेण कति ते यानर्जुनः सन्दर्ध ॥LXXVI ॥ - Ref[(3) p.74]
- Lilavati: (after writing down the problem hurriedl)y on a palm leaf.) Appaji, the problem you have just composed is ' to find the number of arrows taken by Arjuna to fight a fierce battle with Karna. One-half of the number of arrows are used to dissuade the attack of Karna, four times the square-root of it to eliminate Karna's horses, six arrows to destroy the sharp edged spears of Karna. Three arrows were used to sever Karna's Royal umbrella, Royal flag and his bow. The only arrow left-out was used to cut-off Karna's head.' Am I right Appaji ?
- Bhaskara: You are right, . . . Can you recite the sloka explaining a method to solve such problems?

Lilavati: I shall try to sing the sloka Appaji. (Sings melodiously)

राग : भागेश्वरी

- गुणघ्नमूलेनयुतस्यराशेर्दृष्टस्य युक्तस्य गुणार्धकृत्या । मूलं गुणार्धन युतंविहीनं वर्गीकृतं प्रष्टुरभीष्टराशिः ॥ LXXI ॥ - [Ref.3. (p.72)]
- Bhaskara: Marvelous, I never knew you could sing so beautifully Can you explain its meaning.

Lilavati: Yes Appaji. (*Thinks*).... The sloka explains a method to find an unknown (x) which is equal to the sum of the product of a multiplier (a) and the square root of the unknown added to an addend (b), [$x = a\sqrt{x} + b$]. The method is to add the square of half the multiplier and the addend, and then to find the square root of the sum. Square of the sum or difference of the last result and half the

	multiplier is the required value of the unknown. Am I right Appaji?
Bhaskara:	Certainly, Can you solve the problem on Karnavadha? .
Lilavati:	Yes Appaji. (<i>thinks</i>) The given problem is to find <i>x</i> in the equation;

$x = 8\sqrt{x} + 20$

Half of the multiplier 8 is 4, and its square is 16, Sum of 16 and the known quantity 20 is 36 and its square root is 6. The sum of this result 6 and half the multiplier 4 is 10 and its square is 100. . . . The number of arrows carried by Arjuna to kill Karna is one hundred. Am I right Appaji ?

Bhaskara: (*with pleasure*) You are right my child. It is getting late for puja, let us move. (*Lilavati touches the feet of Acharya as a mark of respect and leaves.*)

Curtain Down

Narration 5:

- Narrator 1: Friend, Bhaskara's ever creative mind was in search of suitable situations for constructing mathematical problems in the form of lyrical slokas.
- Narrator 2: His poem on the last battle between Karna and Arjuna is really marvelous. Has he not developed any new concept.
- Narrator 1: Why not ! His *Cakravala* method is acclaimed as one of the best contributions of Indian mathematicians.
- Narrator 2: I didn't understand. Will you please explain?
- Narrator 1: *Brahmagupta* gave a method called *Bhavana* to find approximate values of an irrational number which could be a square number only when 4 or 2 or 1 is added to or subtracted from it. Bhaskaracarya gave a general method for obtaining approximate value of any irrational number.
- Narrator 2: Thank you for the information . . . Acharya is seen Composing something, come on, let's move.

Scene 5

(Late night. Bhaskara is seated brooding over something. He thinks for a while and starts writing. He is unaware of the room getting dark due to the oil in the Nandadeepa being burnt out. The lady enters silently and observes room getting dark. She goes inside and returns with a lighted Nandadeepa.)

- The Lady: *(entering)* What a concentration . Did'nt you notice room getting dark. . . . What are you writing in this dim light ? (*Keeps the nandadeepa by the side of Acharya and peeps at what he is writing*) should you not ask for a light? . . . Are you listening to me?
- Bhaskara: (Finishes writing) I am neither blind nor deaf. I am listening to you. (Hands over the palm leaf just written) This evening, I saw a few swans swimming in the lake, while I was sitting on its bank Since then, I was tempted to write a poem, but I could not find time. I could put it down now Please sing this in Raag Kalavati. (Pleads). Please sing in your melodious voice.

(The lady looks at the sloka for a while smiles mischievously and sings melodiously).

Raag Kalavati

 जातं हंसकुलस्य मूलदशकं मेघागमे मानसम् प्रोइडीयस्थलपद्मिनीवनमगादष्टांशकोऽम्भस्तटात् । बालेबालमृणालशालिनीजले केळिक्रियालालसम् दृष्टं हंसयुगत्रयञ्च सकलां यूथस्य संङ्ख्यां वद ॥ LXXV ॥ – [Ref..3 (p. .22)]

The Lady: Very fine, . . . Acharya is turning romantic now a days.

Bhaskara: Why do you say so?.. What is there in it?

The Lady: Ask me, what is not there in it. Do you think I didn't follow its meaning? . . . Listen "A certain number of swans were swimming in a pond. Ten times the square-root of that number of swans flew towards the *Manasa lake*, one-eighth of it flew towards the forest full of lotus flowers and remaining three pairs were found indulged in an amorous game. The problem is to find the number of swans present in the beginning.

Bhaskara: I never knew that you could analyze a mathematical poem.

(He extends his hand to receive the palm leaf from the Lady. When she bends to hand over the palm leaf, the pearl necklace in her neck, swings in front, and it gets caught between the fingers of Acharya holding the palm leaf. When the Lady stands erect by a jerk, the pearl necklace breaks, and the pearls spill all around. Acharya keeps the palm- leaf on the writing pad and helps the Lady in collecting the pearls. Aftera-while, stands brooding over.)

Bhaskara: ... *haara* ... *haarasthaara* ... *haara sthaarastharunya*... Fine, I shall put it in writing.

A Skit on *Bhaskaracarya* – *II* (12th c. A.D.)

(Acharya sits and starts writing. After collecting all the pearls, the Lady comes near Acharya and looks at what he is writing.)

The Lady: *(teasing)* Should you break my precious pearl necklace to compose this poem *(curiosity)* Am I permitted to see what you have written now?

(Acharya finishes writing and hands over the palm leaf. The Lady reads the poem silently.)

Bhaskara: I shall consider myself blessed if you please sing this poem. (*Pleads*) would you please?

The Lady: You are a champion of flattery, alright, listen.

Raag ; Hindola 8. हारस्तारस्तरुण्या निधुवनकलहे मौक्तिकानां विशीर्णो भूमौ यातस्त्रिभागः शयनतलगतः पञ्चमांशोस्य दृष्टः । प्राप्तः षष्टः सुकेश्या गणक दशमकः संगृहीतः प्रियेण दृष्टं षट्कञ्चसूत्रे कथय कतिपयैमौक्तिकैरेषहारः ॥ LVI ॥ – [Ref. .3 (p. .58)]

(Sarcasm) well written . . . excellent theme to propose a mathematical problem . . .

Are you going to give this problem to Lilavati?

- Bhaskara: I don't understand, . . What has happened to you to night? (*Smiling*) would you please let me know the meaning?
- The Lady: Do you ever ask your daughter to find the number of pearls in a necklace, worn by an young lady severed during an amorous play, if one-third are found on the floor, one-fifth on the bed, one-sixth on her, one-tenth on her lover and remaining six hanging in the necklace'?
- Bhaskara: Of course, I am not giving this problem to Lilavati. . . How could I stop jotting down, the thoughts coming to my mind?. (*With pleasure*) I never knew you could give meaning of Sanskrit poems so well.
- The Lady: (proud) What did you think of me? . . . Am I an ordinary woman? . . . The reputed Lady of the famous mathematician, the Royal Astronomer and the renowned poet Bhaskaraacharya. . . (Dejection) what is the use? . . . (Sobs)

I am not fortunate enough to see my only daughter married.

Bhaskara: What is the urgency now? . . . She has not yet finished her education.

- The Lady: An eight year old girl is ready for marriage . . . But Lilavati is thirteen now . . Do you know how people scorn at us for keeping a grown up daughter not being married?
- Bhaskara: Why do you worry about what others say? . . . Our aim is to see our child happy.
- The Lady: So long, I was also consoled by the same thought.... But now-a-days,

Bhaskara:	(anxiety) Now a days, What happened now?	
The Lady:	She is a grown up girl All her friends got married many years before	
Bhaskara:	I know it (eagerness) Did Lilavati say so ?	
The Lady:	She is my daughter I can realize her feelings You are not at all worried	
	about these mundane matters Do you remember anything in the world other	
	than your observatory and mathematics?	
Bhaskara:	(smiles with hurt feeling) This is what you think of me hm, anyway, I have	
	to contact the bridegroom's father tomorrow.	
The Lady:	(surprise) Bridegroom's father! Who is the Bridegroom?	
Bhaskara:	Ahobalacharya, cousin of our Sayanacharya I forget to tell you Please don't	
	mistake me.	
The Lady:	Why should I? Is this the first time? (Sadness) But, what about the	
	prediction?	
Bhaskara:	The prediction, (consoles) why do you worry about that? Don't you know	
	the will-power could overcome any hurdle in life God is there to bless us all .	
	Have faith in Him,	
Curtain Down		

Curtain Down

Narration 6 :

- Narrator 1: The prediction of astrologers about Lilavathi always gnawed the mind of sensitive Acharya. He found solace in his works on astronomy and mathematics. He started compiling the works of his predecessors and also improving upon their works.
- Narrator 2: You have said, there are lot more to be known about Cakravala. Could you tell me some of them.
- Narrator 1: Bhaskaracharya II developed a generalized process extending Brahmagupta's *Bhavana*, to find rational approximation of any irrational integral number. In the generalized process *Bhavana* is to be repeatedly applied till 'h' is either (± 4) or (± 2) , and therefore the process is named *Chakravala* or Cyclic.
- Narrator 2: But I have heard the *Chakravala* was known even before Bhaskaracharya.
- Narrator 1: It is true. During nineteen sixtees, renowned professor K. V. Shukla found a manuscript attributed to Udayadivakara. It was a commentary on a work of Acarya Jayadeva who lived during 11th Century AD.
- Narrator 2: Acarya Jayadeva ! ... Was he the author of *Gita Govinda*.

- Narrator 1: No, he was a mathematician. Udayadivakara has mentioned Jayadeva's *Cakravala* method. Jayadeva's method slightly differs from that of Bhaskaracharya.
- Narrator 2: I have read the ovation spelt out by a German Mathematician Prof.Hankel about *Cakravala*. Prof.Hankel has said "It is beyond all praise; it is certainly the finest thing achieved in the theory of numbers before Lagrange".
- Narrator 1: Indian mathematicians tried to find the rationale for the *Cakravala* method using Lagrange's method of simple recurring continued fraction expansions of \sqrt{N} . Prof. Clas Olof Selenius of Upsala University, Sweden, developed a continued fraction expansion for \sqrt{N} using all the conditions mentioned in *Cakravala* and proved *Chakravala*. He named his continued fraction as Ideal Continued fraction.
- Narrator 2: I have read an article "Comparative study of Cakravala, MCF expansion method & ICF expansion method of recurring continued fractions of \sqrt{N} for solving

$$Nx^2 + 1 = y^2$$

for integral values for x and y" The article was published by Ramanujan Mathematical Society. What is meant by ICF & MCF expansions?

- Narrator 1: ICF expansion is Ideal Continued Fraction expansion of \sqrt{N} developed by Prof. Selenius of Upsala University, Sweden to prove *Chakravala*. MCF expansion is Modified Continued Fraction expansion developed by Mr.Venkatesha Murthy, with the same intention, during 1963.
- Narrator 2: Mr.Venkatesha Murthy ! . . . Is he the same, who compiled the drama we are presenting now.
- Narrator 1: Yes, you are right. Sorry, we are late. . . As promised to his wife Kalavati, Bhaskaracharya contacted Sayanacharya and got his approval to the marriage of his son with Lilavati. They fixed an auspicious time for the wedding. Bhaskaracharya thought of preparing a water clock to reckon precisely the auspicious time for the wedding. . . Oh, Acharya seems to be busy in the preparation of a water clock. Come on, let us see.

Scene 6

A Skit on **Bhaskaracarya – II** (12th c. A.D.)

(Late in the night, Bhaskaracharya is absorbed in preparing a water clock. The room is lit by lighted Nandadeepa.. A pendulum is kept on a table. By the side of the table, two earthen pots are kept, one above the other in a tripod stand. Bhaskara is observing the bottom of the third earthen pot by holding it above the level of his eyes. Lilavati enters.)

Lilavati: Appaji, ... What are you doing in this late hour?

(Acharya keeps the pot on the table, goes near the pendulum and oscillates it. He counts the number of oscillations by counting the pulse-rate holding his fingers on his wrist nerves.)

Bhaskara: Aren't you getting sleep my child?

- Lilavati: Of late, I am not able to sleep. (Shows interest) What are you finding out Appaji?
- Bhaskara: The period of oscillation, . . . to prepare a water-clock a water clock, to measure the time precisely.
- Lilavati:Water clock (Morose) The water clock, to decide my future. . . . (Sobs) Appaji,I am the cause of all your trouble. I hate myself for bringing you to this plight.
- Bhaskra: (consoles) No, You should never think so. You are my treasure, the only comfort in my life. ... You should never say so. (To change the topic) Do you remember our visit to Karnataka?

Lilavati: Very vaguely Appaji, ... I was just five year old then.

Bhaskara: Don't you remember any interesting spot?

- Lilavati : Yes, .. I remember a beautiful art gallery. ... Was it an art gallery?, No. ... It was a temple. ... A temple resembling an art gallery, ... (*thinks*) was it a temple? ... It must be an art gallery resembling a temple. ... Oh, I am confused. ... What was it Appaji?
- Bhaskara: It is a temple, which is in no way inferior to any renowned art gallery. ...Magnificent temple built under the patronage of Vishnuvardhana, the king of Hoysala. Do you remember to have seen anything peculiar in that temple?
- Lilavati: Is it not a temple of Lord ChennaKeshava, Appaji. ... Many miniature stone carvings decorate the walls around the temple. ... A peculiar thing!. (*Thinks*) Yes Appaji, now I remember. ... it is an idol of Shiva ... An idol of Shiva having ten hands and ten attributes one in each hand.
- Bhakara: You are right; the idol is in the southwestern corner of the temple. ... Do you remember the ten attributes adorning the idol?
- Lilavati: I shall try Appaji, ... (*recollects*) Pasha, ankusha, ahi, damaruka, kapala, shoola, ... shoola, ... shoola, What are the other four ? ... shoola, shoola ... Oh no, I am not getting Appaji ?... What are the other four?

Bhaskara: Khatwanga, shakti, shara, chapa (stops working, thinks aloud)

9. पाशांड्कुशाहिडमरूककपालशूलैः खट्वाङ्गशक्तिशरचापयुतैर्भवन्ति । अन्योन्यहस्तकलितैः कतिमूर्तिभेदाः शम्भोर्हरेरिवगदारि सरोज शंङ्खैः ॥ CCLXX ॥ [Ref. 3 (p.178)]
Lilavati: (Listens attentively) Can I sing this Appaji,
Bhakara: (Looks at her with pleasant surprise) Excellent, you can remember the sloka

hearing it once and set raga also to it. Wonderful, sing my child, ... sing.

Lilavati: With your blessings Appaji, (Bows with respect and sings)

Raag : Hamsanandi

पाशांङ्कुशाहिडमरूककपालशूलैः खट्वाङ्गशक्तिशरचापयुतैर्भवन्ति ।

अन्योन्यहस्तकलितैः कतिमूर्तिभेदाः

शम्भोर्हरेरिवगदारि सरोज शंङ्खेः || CCLXX || [Ref. 3 (p.178)]

Bhaskara: Will you please tell its meaning.

Lilavati: *(with pleasure)* The problem you have just composed is on permutation. It is about finding the number of variations of Shiva, when these ten attributes are interchanged reciprocally in all the ten hands. And also to find the number of variations of Lord Vishnu having four hands and four attributes one in each hand, when they are interchanged similarly in all the four hands. Am I right, Appaji?

Bhaskara: You are absolutely right. Do you remember the rule to solve such problems?

Lilavati: I shall try, Appaji. The rule is (*Recollects, and then recites*)

स्थानान्तमेकादि चयाङ्क घातः सङ्ख्याविबेधा नियतैःस्युरङ्कैः ॥267॥

[Ref. 5 (p.232)]

Bhaskara: Can you solve the problem

Lilavati: I shall try, Appaji. To find the number of variations, we have to find the product of all natural numbers from 'one' to 'the number of attributes, ten.'. ... so, the number of variations of Lord Shiva is the product of all natural numbers from one to ten. ... It is ... one into two is 2, two into three is 6, six into four is 24 ... (*multiplies mentally*) The number of idols of Lord Shiva is thirty six lakhs twenty eight thousand eight hundred (36,28,800) Appaji. ... The number of variations of Lord Vishnu is twenty four. Am I right Appaji?

A Skit on **Bhaskaracarya** – II (12th c. A.D.)

(Bhaskara, who had stopped working to hear Lilavati solving the problem, exclaims with pleasure.)

Bhaskara : गर्वितगणकबह्नां स्यात्पातो अवश्यम् अडकपाशेस्मिन् ॥277॥

When most of those, proud of their calculating skill, invariably fumble while solving problems on permutations, the Ankapasha; you have acquired mastery over it. ... I am proud of you my child, I am proud of you.

Lilavati: It is all your blessings Appaji. ... Am I not your sincere disciple and affectionate daughter? ... Appaji, I am fortunate to be near you, whenever you composed these lyrical mathematical poems. Revered Sayanacharya is never tired of praising your works.

Bhaskara: It is his affection, ... (*broods over*) My works. ... Are they really? ... To tell you the truth

Raag Revathi

10. ब्रहमाहवयश्रीधरपग्ननाभबीजानि यस्मादतिविस्तृतानि । आदायतत्सारमकारि नूनं सद्युक्तियुक्तं लघुशिष्यतुष्ठैः ॥2॥[Ref. (1) (p.381)]

- Lilavati: Your greatness is manifested in this poem Appaji. Other-wise, who admits that "the stupendous works of Brahmagupta, Shridhara and Padmanabha are abridged by me and they are presented in the concise form, with new techniques here and there, for the benefit of the students." ... , Appaji, have you thought of any title for your great work?
- Bhaskara: Great Work. ... What is great in it? ... I am not sure. ... But I am sure on one thing. ... Joy and happiness indeed ever increases for those who have *Lilavati* as a charming woman closely embraces, whose person is embellished by an assemblage of elegant qualities, who is pure and perfect in her conduct and who utters agreeable discourse.

(Composes a poem and sings)

Raag Sindhubhairavi

 येषां सुजातिगुणवर्गविभूषिताङ्गी शुद्धाखिलव्यवहृतिः खलु कण्ठसक्ता । लीलावतीह सरसोक्तिमुदाहरन्ती तेषां सदैव सुखसम्पदुपेति वृद्धिम् ||CLXXVIII|| [1 (p.181)]

Lilavati: (*smiles mischievously*) Appaji, you have just now named your work.

Bhaskara: I couldn't follow you, what do you mean my child?

- Lilavati: The poem you have just now composed, may also mean 'Joy and happiness is indeed ever increasing for those who have *Lilavati* clasped to their throats (by constant repetition Of the text), decorated as the members are with neat reduction of fractions, multiplication and involution, pure and perfect as are the solutions, and tasteful as is the speech which is exemplified.'
- Bhaskara: (appreciates) You are absolutely right. ... But I never thought so. ... The title LILAVATI would aptly suit to my work on Arithmetic and Mensuration. ... Lilavati, you are now fully educated. ... May God bless you with all prosperity in your life. (Lilavati touches his feet with respect)

(As the curtain slowly closes, the sloka below is sung in chorus from the background.

Raaga: Revathi

12. येषां सुजातिगुणवर्गविभूषिताङ्गी शुद्धाखिलव्यवहृतिः खलु कण्ठसक्ता । लीलावतीह सरसोक्तिमुदाहरन्ती तेषां सदैव सुखसम्पदुपेति वृद्धिम् ॥CLXXVIII॥ Curtain Dow

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